Closing the loop

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Closing the loop: optimal strategies for hybrid manufacturing / remanufacturing systems

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To my husband Turan

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Contents

Chapter 1

Introduction

This chapter provides an overview of closed-loop supply chains and remanufacturing activities. A discussion of challenges in remanufacturing industry is presented, which motivates the work in this thesis.

The global human population, prior to the Industrial Revolution, was around 300 million at A.D. 1000, 500 million at A.D. 1500, and 790 million by A.D. 1750. The twentieth century has witnessed extraordinary population growth where the world population increased from 1.65 billion to 6 billion and it is expected to be 9 million or more by the mid-twenty-first century (United Nations, 1999).

The level of consumption by this growing population has also been continuously increasing. As a result, the world now faces serious environmental and economical problems such as depletion of natural resources, shortage of key minerals and metals, and waste with presence of toxic materials (Pochampally et al., 2009).

For instance, in the EU, E-waste or Waste Electrical and Electronic Equipment (WEEE) increases by 16-28% every five years, which is three times faster than average annual municipal solid waste generation (United Nations Environment Programme, 2007). In the US, it is estimated that over 375 million empty toner cartridges and ink cartridges are thrown away as scrap every year and most of these cartridges end up on landfill sites or in incinerators (Vasudevan et al., 2012). Moreover, according to Environment Protection Agency (EPA), in 2009 about 2.37 million tons weighted TVs, computers, peripherals (including printers, scanners, fax machines) mice, keyboards, and cell phones were discarded. All these electronic products are made from valuable resources, including precious and other metals, engineered plastics, glass, and other materials, all of which require energy to source and manufacture.

As we are more exposed to these environmental issues, environmental consciousness has also grown. The term sustainable development first stated in 1987 by the World Commission on Environment and Development as "*a development that meets the needs of the present without compromising the ability of future generations to meet their own need*" (WCED, 1987). For sustainable development, in 2007, EU member states committed to a 20-20-20 initiative by 2020 which aims at a 20% increase in energy efficiency, a 20% reduction in global warming gas emissions, and the generation of 20% of its energy needs with renewable forms of energy by 2020 (based on 1990 levels). Also, in 2003, to reduce e-waste, the EU enacted WEEE directives that set collection, recycling and recovery targets for all types of electrical goods. And in the US, there are more than 25 states with some form of take-back legislation for e-waste (Souza, 2013).

1.1 Closed-loop supply chains

Due to these legislative activities and increasing consumer awareness closed-loop supply chain management and product recovery are becoming more important for corporations. Closed-loop supply chain management is defined as the design, control, and operation of a system to maximize value criterion over the entire life-cycle of a product with dynamic recovery of value from different types and volumes of return over time. (Guide and Van Wassenhove, 2009). It is stated that a key to a corporation being environmentally responsible is minimizing waste and recovery of materials used for manufacturing. This can prevent of waste and avoid environmental costs before they occur. Material recovery systems include strategies such as repairing, remanufacturing and recycling products (Guide, 2000).

Currently, recycling is the most commonly used strategy for product in the US (Giuntini and Gaudette, 2003). For simple items such as beverage containers, steel products and paper goods, recycling is a highly effective strategy. However, recycling a more complex product like a car or a cellular phone results in a loss of up to 95 percent of the value added to the product. Remanufacturing on the other hand, which can be interpreted as the ultimate form of recycling, conserves not only the raw material content but also much of the value added.

Remanufacturing is defined as the process of bringing used products to a "like-new" functional state with warranty to match. Today, remanufacturing is practiced in various sectors such as consumer electronics, toner cartridges, household appliances, machinery, cellular phones, aerospace industry, etc. In these various sectors, although there may be differences in remanufacturing activities, there are three primary groups of activities in every case, namely, (1) product returns management, (PRM) (2) remanufacturing operational issues, and (3) remanufactured products market development (Guide and Van Wassenhove, 2006). PRM includes product acquisition management processes and focuses on the timing, quality, and quantity of returned products and is defined as the front end of remanufacturing activities. Remanufacturing operational issues include reverse logistics; test, sort, and disposition activities; product disassembly and remanufacturing processes. Remanufactured marked development which is defined as the back end of remanufacturing activities, includes remarketing activities, channel choice, coordination, and market cannibalization issues.

1.2 Benefits of remanufacturing

Remanufacturing has numerous benefits for the environment and economies. In the UK, remanufacturing is estimated to have a value of £5 billion and to represent UK-wide savings of 270,000 tones of raw materials and 800,000 tones of CO_2 (Parker and Butler, 2007). Also, on a life cycle basis, remanufacturing photocopiers consumes 20-70% less materials, water, and energy and generates 35-50% less waste than conventional manufacturing (Toffel, 2004). For the original equipment manufacturers (OEMs), remanufacturing provides: savings in labor, material and energy costs; shorter production lead times; balanced production lines; new market development opportunities, and a positive, socially concerned image (McConocha and Speh, 1991). For instance, OEMs that adopt remanufacturing are estimated to save 40 to 60 percent of the cost of manufacturing a completely new product and in the 2008/2009 financial year, Fuji Xerox Australia remanufactured more than 230,000 equipment parts, equating to a $6 million cost-saving compared to sourcing new part (Dowlatshahi 2000). Additionally, adopting remanufacturing as a part of the production strategy allowed Caterpillar to create a new market among contractors who cannot afford to buy a Caterpillar product outright (Gutowski et al., 2001). IBM Europe and Xerox have reported that their product recovery activities have strengthened their brand image. There is also an encouraging market for remanufactured products in the US. Already in 1997, approximately 73,000 US firms had sold an estimated $53 billion worth of remanufactured products such as automotive parts, cranes and forklifts, furniture, medical equipment, pallets, personal computers, photocopiers, telephones, toner cartridges etc. (Ferguson and Toktay, 2006).

1.3 Demand cannibalization

In the previous section, the potential benefits of remanufacturing were described. Despite all of these benefits, firms often have reservations about adopting remanufacturing. One of the major concern is that when the remanufactured product is sold on the same market as the new market, it attracts the same customer population and thereby cannibalizes the sales of the new products.

The degree of (potential) cannibalization depends on the comparability of the new and remanufactured product. When remanufactured products are perfect substitutes for new products, then each sale of a remanufactured product implies one less sale of a new item, and so remanufacturing is mainly used to reduce (re)manufacturing costs and increase profit margins. There exist some product categories such as single-use cameras and refillable containers where remanufactured items are indistinguishable from the new product. Also, in Japan Fuji Xerox incorporated reused components in new products without labelling them as containing remanufactured components (Matsumoto and Umeda, 2011). In all these cases, consumers perceive no distinction between the remanufactured and new items, i.e., remanufactured items are perfect substitutes for newly manufactured products. However, most of the remanufactured products are usually sold at discounted prices and consumers can differentiate between a new and a remanufactured product. For instance, Hewlett-Packard offers remanufactured (refurbished) computers with 40% discount and Dyson sells remanufactured vacuum cleaners with prices $100 below the price of a new one. When a remanufactured product is sold at a discounted price, cannibalization becomes a concern and firms consider this situation undesirable, especially when the margin on the new product is higher than the margin on the remanufactured product. Thus, it is important to jointly determine the prices of the new and remanufactured products to optimize the profit.

1.4 Competition and cooperation in remanufacturing

External competition is another major issue for firms. Independent Operators (IOs) can compete for returns and a share of the market for remanufactured products. In this case, OEMs face external cannibalization rather than internal cannibalization, or both if the OEM itself also remanufactures products. For instance, it is estimated that 95% of the remanufacturing programs are not managed by the OEMs (Guide, 2000) and independent operators have a clear dominance in the mobile phone and automobile parts remanufacturing industry. Moreover, it is stated that it is very difficult for OEMs to enter the remanufacturing market after it becomes dominated by IOs (Atasu et al., 2010).

Apart from competition, cooperation is also possible between the agents in a closed-loop supply chain, especially between those responsible for collecting the used-products. For instance, some companies may prefer to collect used items directly from the customers but others may prefer independent third parties (e.g., Genco) to handle used product collection. Also, in the auto industry, independent third parties (i.e., dismantlers) are handling used-product collection activities for the OEMs. Moreover, cooperation between OEMs is also possible. Blinsky (1995) points out that the "big three" auto manufacturers in the US, namely Chrysler, GM and Ford, have started to invest in joint research and remanufacturing partnerships with dismantling centers to benefit from scale economies and shared experience. Also, in 1990, Kodak established alliances with Fuji and Konica at the collection facilities to collect returned single-used cameras and in Japan, Fuji Xerox, Ricoh and Canon have formed partnerships to collect and return each other's used photocopier machines. Six printer ink and toner ink cartridge OEMs (Epson, Canon, Hewlett-Packard, Brother, Dell, and Lexmark) have collaborated to collect used ink cartridges (Matsumoto and Umeda, 2011).

1.5 Research objectives and questions

Potential benefits of remanufacturing were presented in the previous section, along with an empirical discussion of remanufacturing practices in various sectors either by OEMs or IOs in both in competitive and cooperative environments. However, most of the firms today do not have a clear understanding for adopting remanufacturing and how to position / price a remanufactured product (Ferguson and Toktay, 2006; Atasu et al., 2008). Motivated by this, the main goal of this thesis is to provide analytical tools to the firms for optimal pricing, manufacturing and remanufacturing strategies. This goal is rather broad and to focus on the research, this thesis addresses four specific research questions. These research questions are described in the following paragraphs.

It is stated that to make remanufacturing economically attractive, one needs adequate quantities of used products of the right quality and price, at the right time, as well as a market for recovered products (Guide and Van Wassenhove, 2009). Accordingly, the first research question deals with two aspects of the remanufacturing activities, namely, product acquisition management and pricing of new and remanufactured products.

What is the optimal quantity and quality of the acquired cores and the price at which new and remanufactured products are sold?

In addition to its presented benefits, remanufacturing may offer a better alternative to capacity constraint on new product manufacturing (Atasu et al., 2008) and the second research question deals with profitability of remanufacturing in a capacitated setting.

What is the impact of remanufacturing on the optimal capacity and production decisions? If remanufacturing is either more costly or more capacity intensive, can it still be profitable?

As it is describe previously, firms can both compete and cooperate in closed loop supply chains. In the third research questions competition is considered.

What are the optimal production and pricing strategies of the parties in a supply chain in the case of competition and what is the effect of competition on the remanufacturing strategy?

In research question four, motivated by the real life examples, cooperation of OEMs on collecting the used items is considered.

When cooperation on collecting used items will be beneficial for OEMs? When does cooperation lead to joint cost-savings, and how should these savings be allocated among the firms?

This thesis is organized as a collection of research papers centered around the main research theme and each chapter consists a research paper that deals with a specific research question listed above. Since the chapters of this thesis are devised also as a research paper that can be read individually, there is some overlap in positioning the chapters. Each individual chapter mathematically defines and formalizes a particular problem and determines the optimal solution to the developed model. The specific and relevant literature for each chapter is found within the chapter itself. In the following section, we provide a brief outline of the thesis and state the contributions of each research paper.

1.6 Thesis outline

The thesis is organized as follows. Chapter 2 addresses the first research question and combines two aspects of remanufacturing activities namely, product acquisition management and marketing (pricing) of the remanufactured products and this simultaneous consideration of product acquisition management and marketing activities is the main contribution to the existing literature. A single OEM is considered in a single period setting and the OEM decides on the acquisition prices offered for returns from different quality types and on selling prices of new and remanufactured products. A procedure is developed to determine the optimal prices and production amount. Also, a sensitivity study is conducted to understand the effect of different model parameters on the optimal strategies and profit.

Chapter 3 deals with the second research question and investigates the effect of remanufacturing on capacity and production decisions. A two-period model with manufacturing in both periods and the option in the second period to remanufacture products that are returned/collected at the end of the first period is analyzed which is inspired by the situation for a specific car company. The main focus is on the setting where remanufacturing is less costly and less capacity intensive than manufacturing. Optimal manufacturing and remanufacturing quantities are derived and it is analyzed under what conditions (specified by costs, capacity restrictions and demand) remanufacturing leads to increased total production. Later, the settings where remanufacturing is either more costly or more capacity intensive than manufacturing is considered and the results are compared to those of the main setting.

In Chapter 4, we consider competition between a single OEM and a single IO and analyze the effect of competition on the optimal production and pricing strategies. Different from the existing literature, in the presented model the OEM and IO compete not only for selling their products but also for collecting returned products (cores) through their acquisition prices. The problem is considered in a two-period setting with manufacturing by the OEM in the first period, and manufacturing as well as remanufacturing in the second period. Non-cooperative game theory is used to find the optimal policies for both players in the second period. Then, the optimal manufacturing decision for the OEM in the first period is determined. To gain insights on the effect of system parameters on the optimal solution and the profitability of remanufacturing an extensive numerical study is conducted.

In Chapter 5, the last question is addressed that considers cooperation between OEMs on collecting the used items. In this chapter, the conditions where it is beneficial for OEMs to cooperate by forming a coalition in setting up a returns network are determined. Using cooperative game theory, an efficient allocation of cost savings between the parties is found.

Finally, in Chapter 6, individual chapters are summarized and possible directions for further research are outlined.

Chapter 2

Optimal core acquisition and pricing strategies for hybrid manufacturing and remanufacturing systems

In this chapter, we combine two aspects of remanufacturing, namely product acquisition management and marketing (pricing) of the remanufactured products. We consider an original equipment manufacturer (OEM) who decides on the acquisition prices offered for returns from different quality types and on selling prices of new and remanufactured products, in a single period setting. We develop a procedure for determining the optimal prices and corresponding profit of the OEM, and conduct a sensitivity study to understand the effect of different model parameters on the optimal strategies and profit. A counter-intuitive and an important managerial result is that, for the optimal solution instead of having the same profit per remanufactured item for all return types, the profit per item should be higher if the total cost for acquisition and remanufacturing is lower.

2.1 Introduction

Sustainable and environmental manufacturing processes have gained increasing attention both from industry and academia over the last twenty years, since industrialization and population growth have increasingly burdened the environment. To reduce this burden, European companies have been made legally responsible for collecting their end-of-life products and adopting the sustainable production strategy of product recovery. Companies in other countries, including the US, are not legally bound to collect used-products, but many still do so because of the economic benefits (Kaya, 2010).

Remanufacturing is one of the product recovery processes and is defined as the process of bringing used products to a "like-new" functional state with warranty to match. Today, remanufacturing is practiced in various sectors such as consumer electronics, toner cartridges, household appliances, machinery, cellular phones, aerospace industry, etc. In each of these sectors, although variations exist in the reverse supply chain design, both products returns management (PRM) and remanufactured products market development play vital roles (Guide and Van Wassenhove, 2006).

PRM, which is the front end of a closed-loop supply chain, includes product acquisition management processes and focuses on the timing, quality, and quantity of returned products. A firm that operates a recoverable product system relies on PRM as the basic input to a reuse system is product returns. A firm has two main options to obtain used products from the end-users: the waste stream system and the market-driven system. In the waste stream system, a firm passively accepts all products returns, which implies high uncertainty in the quality of returns. However, with a market driven system where end-users are motivated to return end-of-life products by financial incentives, firms can control the quality level of return products (Guide and Van Wassenhove, 2001). For instance, Dell, Apple, and ReCellular provide a schedule of prices across various quantities and qualities of used products. Also, Robert Bosch GmbH installed electronic data logs in their power tools to identify the residual quality of used products and similar data logs are stated to be developed for other products such as large household appliances (white goods) (Toffel, 2004). In this study, we focus on a market-driven system that proactively sources used products at the optimal price and quality, as this has been shown to be an important driver of the profitability of a closed-loop supply chain (Guide and Van Wassenhove, 2009).

The back end of the closed-loop supply chain processes is the market development of remanufactured products which includes remarketing activities, channel choice and coordination, and market cannibalization issues. There exist some product categories such as single-use cameras and refillable containers where remanufactured items are perfect substitutes for newly manufactured products. To the contrary, remanufactured mobile phones and PCs are sometimes sold to completely different secondary markets. However, in most situations, markets partially overlap and remanufactured products are sold at reduced prices. This explains why many firms have reservations about adopting remanufacturing as a business strategy and particularly fear internal cannibalization (Guide and Van Wassenhove, 2009). Firms consider this situation undesirable especially when the margin on the new product is higher than the margin on the remanufactured product. Thus, it is important to jointly determine the prices and production quantities of new and remanufactured products in order to optimize the profit. However, as Ferguson and Toktay (2006) and Atasu et al. (2008) state, most of the firms today do not have a clear understanding for adopting remanufacturing and how to position / price a remanufactured product.

In this chapter, we consider both front end decisions on product acquisition management and back end decisions on the marketing (pricing) of remanufactured products. Specifically, we optimize the number and quality of the acquired cores and the price at which remanufactured products are sold. The main contribution of the chapter is that we simultaneously consider both types of decisions, whereas the existing literature consists of two (almost) separate streams, as discussed in the next section. A counter-intuitive and an important managerial result is that, for the optimal solution instead of having the same profit per remanufactured item for all return types, the profit per item should be higher if the total cost for acquisition and remanufacturing is lower.

The rest of the chapter is organized as follows. The next section reviews the related remanufacturing literature on product acquisition management and joint pricing of the new and remanufactured products. Section 2.3 describes the system that we analyze. In Section 2.4, we develop the algorithm for finding the optimal solution, and in Section 2.5 we conduct a sensitivity analysis and a numerical study to understand the effects of the system parameters on the optimal solution. Finally in Section 2.6, a brief summary of the findings and managerial insights are provided, and avenues for further research are discussed.

2.2 Related literature

There are numerous studies on closed-loop supply chains and remanufacturing in the current literature. Early studies reviewed by Fleischmann et al. (1997) and Guide and Van Wassenhove (2009) describe the evolution of the research on closed-loop supply chains. Tang and Zhou (2012), Junior and Filho (2012) and Souza (2013) also provide extensive reviews of more recent studies.

In this chapter, we focus on a hybrid manufacturing and remanufacturing system in the presence of different return types. Aras et al. (2004) and Ferguson et al. (2009) study the value of categorizing returned products for such a system. Aras et al. (2004) assume that the newly manufactured and remanufactured items are perfect substitutes and there is no capacity restriction for remanufacturing. Ferguson et al. (2009) relax these assumptions and examine the potential benefits for quality grading of returns. Different from these studies, we also focus on product acquisition management where the quality of the returns depends on the acquisition prices.

Product acquisition management has been widely considered in the remanufacturing literature. Guide and Van Wassenhove (2001) develop a framework for analyzing the profitability of reuse activities and show the impact of product acquisition management on the profitability of reuse activities. Based on this approach, Guide et al. (2003) build a mathematical model by considering a remanufacturer in a single period where used products in multiple quality categories (affecting the remanufacturing cost that is assumed to be fixed and known for each category) are acquired from third-party brokers, and they determine the optimal selling price and the acquisition prices that maximizes the profit rate. Galbreth and Blackburn (2006) relax the assumption that the remanufacturing costs are fixed and allow for scrapping of some of the acquired returns. They determine the optimal acquisition and sorting policies in the presence of used product variability. Bakal and Akcali (2006) study the effect of random yield in remanufacturing, where the yield depends on the acquisition price offered for used products. They determine the optimal acquisition price for the end-of-life products and the selling price for remanufactured parts. Vadde et al. (2007) consider a product recovery facility and determine the optimal pricing strategy of reusable and recyclable components in the presence of legislation that limits the disposal quantity. They consider both a single and multiple types of product returns and compare a proactive acquisition policy with a passive policy. All these studies focus on a pure remanufacturing, whereas our focus is on a hybrid manufacturing and remanufacturing system.

Minner and Kiesmüller (2012) also consider a hybrid manufacturing and remanufacturing system in a dynamic setting where demand can be satisfied with either newly manufactured products or remanufactured products. They assume that demand is not price dependent and there is no difference between the new and remanufactured products. Instead, we analyze a model where new and remanufactured products are sold to different prices and these prices affect the demand.

Front end decisions have received less attention in the remanufacturing literature, although several assumptions about consumer's perception and willingness to pay for remanufactured products have been considered. On the one hand, some researchers assume that consumers see no difference between new and remanufactured products such as Majumder and Groenevelt (2001), Savaskan et al. (2004), Ferrer and Swaminathan (2006), and Geyer et al. (2007). In these studies, remanufactured products are assumed to be the perfect substitutes for the new products. On the other hand, some researchers assume that the market for remanufactured and new products are fully segmented and there is no cannibalization. In the middle, some researchers assume that there exists partial cannibalization and remanufactured products competing with new products on the basis of price, e.g., Ferguson and Toktay (2006), Debo et al. (2005), and Atasu et al. (2008). In all these studies, the consumers' willingness to pay for a remanufactured product is assumed to be heterogeneous, uniformly distributed on a given scale, and lower than that of a new product. We will take a similar approach.

Our main contribution to the existing remanufacturing literature is that we simultaneously consider product acquisition management and pricing / positioning of remanufactured products.

2.3 The model

We consider a single period model for a single OEM. For the core acquisition side of the modeling, we adopt a similar model used by Guide et al. (2003). We assume that there is no constraint on the supply of the remanufactured products. Also, there is no setup cost for remanufacturing and no capacity constraint for manufacturing new products or remanufacturing acquired cores.

There are n different return types with respect to remanufacturability and minimal acquisition price. We denote c_i as the unit remanufacturing cost of return type i and Ψ_i as the minimal acquisition price below which there will be no returns for type i, and assume without loss of generality that $\Psi_1 + c_1 \leq \Psi_2 + c_2 \leq ... \leq \Psi_n + c_n$ holds. Since $\Psi_i + c_i$ is the minimum total cost at which a core of type i can be acquired and remanufactured, we will refer to this order as the preference order, where type 1 is the most preferred type and type n is the least preferred. The return rate of type i is a linear function of the acquisition price, a_i, given by $R(a_i) = \alpha_i(a_i - \Psi_i)$. All return types are assumed to be remanufactured up to the same standard and sold at the same price which is defined as p_r. The total amount of remanufactured product sales is denoted by q_r and the amount of new product sales by q_0. We also assume that the number of products manufactured by the OEM is positive, i.e., $q_0 > 0$, which holds in most real-life settings. The sales price for new products is defined as p_0 and unit manufacturing cost of new product is defined as c_0.

Remanufactured products are distinguishable from new ones, and consumers have lower willingness to pay for remanufactured products. For the inverse demand functions, we adopt the same modeling as Debo et al. (2005) and Ferguson and Toktay (2006) where

$$p_0 = 1 - q_0 - \delta q_r, \tag{2.1}$$

$$p_r = \delta(1 - q_0 - q_r). \tag{2.2}$$

The reasoning behind (2.1) and (2.2) is explained in detail in Appendix 2.A. A list of notations is given in Table 2.1.

It is obvious that it will not be optimal for the OEM to acquire (pay for) cores which will not been remanufactured. Thus, in the optimal solution, all collected cores must be remanufactured, i.e.,

$$q_r = \sum_{i=1}^{n} \alpha_i(a_i - \Psi_i). \tag{2.3}$$

Model Parameters	
n	Number of different return types
c_0	Cost of manufacturing per new product
c_i	Cost of remanufacturing per return type i, $i = \{1, .., n\}$
Ψ_i	Minimal acquisition price of return type i, $i = \{1, .., n\}$
α_i	Return rate coefficient of type i, $i = \{1, .., n\}$
δ	Consumers' relative willingness to pay for remanufactured product
(Decision) Variables	
p_0	Sales price of new products
p_r	Sales price of remanufactured products
a_i	Acquisition price offered per return type i, $i = \{1, .., n\}$
q_0	Number of newly manufactured products
q_r	Number of remanufactured products
$R(a_i)$	Number of collected returns of type i, $i = \{1, .., n\}$
q_i	Number of remanufactured returns of type i, $i = \{1, .., n\}$
q_{tot}	Total production amount
m^*	Number of return types optimal to purchase
Other notations	
Π	OEM's profit
$\overline{x}$	Denotes optimal solution for the unconstrained problem
$\overline{x}^k$	Denotes optimal solution for the unconstrained problem when there are k types available
x^*	Denotes optimality, e.g., p_n^* is the optimal selling price of new products

Table 2.1: Notations

Then, the objective function becomes,

$$\begin{aligned}
\max \Pi &= \max_{(q_n, a_1, .., a_n)} (1 - q_0 - \delta q_r - c_0)\, q_0 \\
&\quad + \delta (1 - q_0 - q_r)\, q_r - \sum_{i=1}^{n} \alpha_i (a_i - \Psi_i)(a_i + c_i) \\
s.t\ \Psi_i &\leq a_i \qquad \forall i = 1, 2, .., n \\
q_0 &\geq 0,\ q_r \geq 0
\end{aligned} \tag{2.4}$$

Proposition 2.3.1 *The objective function* Π *is jointly concave in terms of the decision variables.*

Proof. See Appendix 2.B. ■

We start our analysis with the unconstrained problem. Since the objective function is concave as stated in Proposition 2.3.1, there exist a unique optimal solution of the unconstrained problem, derived by simultaneously solving the first order conditions. Proposition 2.3.2 presents the optimal solution of the unconstrained problem.

Proposition 2.3.2 *For the unconstrained problem in (2.4), where there are n return types available, the optimal values of the decision variables are*

$$\overline{a}_j^n = \frac{\delta c_0 + \delta(1-\delta)\left(\sum_{i=1}^{n} \alpha_i (c_i + \Psi_i + \Psi_j - c_j)\right) + \Psi_j - c_j}{2\left(1 + \delta(1-\delta)\sum_{i=1}^{n} \alpha_i\right)} \qquad \forall j = \{1, .., n\}, \tag{2.5}$$

$$\overline{q}_r^n = \frac{\delta c_0 \sum_{i=1}^{n} \alpha_i - \sum_{i=1}^{n} \alpha_i (c_i + \Psi_i)}{2\left(1 + \delta(1-\delta)\sum_{i=1}^{n} \alpha_i\right)}, \tag{2.6}$$

$$\overline{q}_0^n = \frac{1 - 2\delta \overline{q}_r^n - c_0}{2}, \tag{2.7}$$

$$\overline{p}_0^n = \frac{1 + c_0}{2},\ \overline{p}_r^n = \delta(\overline{p}_0^n - (1-\delta)\overline{q}_r^n). \tag{2.8}$$

Proof. See Appendix 2.B. ■

Note that the optimal values of the decision variables for the unconstrained problem when there are k return types available, where $1 \leq k \leq n$ holds, are similar to those of presented in Proposition 2.3.2.

From (2.8) we observe that $\overline{p}_r \leq \delta \overline{p}_0$ must hold to ensure non-negative sales price for the remanufactured items. The number of remanufactured products is non-negative, i.e., (2.7) is non-negative when

$$\sum_{i=1}^{n} \alpha_i (\delta c_0 - c_i - \Psi_i) \geq 0 \tag{2.9}$$

holds. Since $\alpha_i > 0$ for all $i = \{1, .., n\}$, (2.9) always holds when the condition $\delta c_0 \geq c_i + \Psi_i$ for all $i = \{1, .., n\}$ holds. This condition can be interpreted as that for which profitable remanufacturing is possible, as otherwise the profit (sales price minus cost) per remanufactured item would be below that of new item.

Also, the amount of newly manufactured products, q_0, will be non-negative when

$$1 - c_0 > \delta \sum_{i=1}^{n} \alpha_i (c_0 - c_i - \Psi_i - 1 + \delta) \tag{2.10}$$

holds. This condition implies that the profit per remanufactured item is not too high compared to profit per new item, so that the OEM continues manufacturing instead of switching fully to remanufacturing. As OEMs typically do not fully switch to remanufacturing, and will only consider remanufacturing if it is profitable, we assume that (2.9) and (2.10) hold in what remains.

Moreover, using (2.5), we find that in the optimal solution of the unconstrained problem the difference between the acquisition prices offered for two different return types only depends on the minimal acquisition prices and remanufacturing cost for those types, i.e.,

$$\overline{a}_k^n - \overline{a}_i^n = \frac{(\Psi_k - \Psi_i) - (c_k - c_i)}{2}. \tag{2.11}$$

This will turn out to be useful for developing a time-efficient algorithm in Proposition 2.3.4.

Due to ordering the classes with respect to the minimum required acquisition and remanufacturing cost plus in line with the findings of Guide et al. (2003), it is intuitively optimal to only purchase in classes $1, .., m$ for some $m \in \{1, .., n\}$. This is formalized in Lemma 2.3.1.

Lemma 2.3.1 *It can never be optimal to acquire cores of some type without purchasing cores from all more preferred types as well.*

Proof. See Appendix 2.B. ■

Lemma 2.3.1 suggests that it only remains to be decided up to which preference class, m, cores will be acquired. Lemma 2.3.2, which considers the effect of the number of available types on the unconstrained (for that number) optimal acquisition prices, is useful to this end.

Lemma 2.3.2 *As the number of types from which core are acquired increases from $m-1$ to m, the difference between the acquisition prices offered to return type j will be*

$$\overline{a}_j^{m-1} - \overline{a}_j^m = \frac{\delta(1-\delta)\alpha_m \left(\overline{a}_m^m - \Psi_m\right)}{\left(1 + \delta(1-\delta)\sum_{i=1}^{m-1} \alpha_i\right)}, \qquad \forall j = \{1, ..., m-1\}.$$

where $\overline{a}_j^{m-1}$ and $\overline{a}_j^m$ are optimal acquisition prices offered for the unconstrained problem for return type j when there are $m-1$and m return types respectively.

Proof. See Appendix 2.B. ■

Note that if the price difference in Lemma 2.3.2, $\bar{a}_j^{m-1} - \bar{a}_j^m$, is negative, then fewer (if any) cores are acquired from return type j,$j \in \{1, .., m-1\}$, if type m is removed. So, if no cores are acquired from type j with m types, then this will continue to be the case after removing type m, or (inductively) removing even more types for $j < m-1$. This is formalized and proven in Proposition 2.3.3.

Proposition 2.3.3 *For a given number of core types, $m \leq n$, if there exists some k such that $\bar{a}_i - \Psi_i < 0$ for $i = k+1, ..., m$, then we never purchase cores of type $i = k+1, ..., m$ in the global optimal solution.*

Proof. Let $\bar{a}_i^m - \Psi_i < 0$ for $i = k+1, ..., m$. Then, Lemma 2.3.2 implies that $\bar{a}_i^{m-1} - \bar{a}_i^m < 0$ for $i = 1, ..., m-1$ and this leads to the condition $\bar{a}_{m-1}^{m-1} - \Psi_{m-1} < 0$ since $\bar{a}_{m-1}^m - \Psi_{m-1} < 0$ holds. Then, by induction, we obtain that $\bar{a}_{k+1}^{k+1} - \Psi_{k+1} < 0$ holds. ■

Proposition 2.3.3 is useful to find the global optimal solution efficiently, as it implies that solutions that acquire cores from all classes up to $j \in \{k+1, .., m\}$ does not need to be considered under the condition provided. Instead, the search procedure can jump from m to k. This leads to the procedure to find the optimal solution that is presented in Proposition 2.3.4.

Proposition 2.3.4 *The optimal prices and production amount are determined by the following procedure.*

Step 0. *Set $m = n$.*

Step 1. *Compute the unconstrained solution with m types of cores by first determining $\bar{a}_1$ from (2.5), then compute $\bar{a}_i$ for $i = 2, .., m$ using relation in (2.11).*

Step 2. *Check $\bar{a}_i - \Psi_i \geq 0$ for $i = 1, ..., m$, where $\bar{a}_i$ is the unconstrained solution. If all m conditions are satisfied, stop, the solution is optimal. If there exist any $\bar{a}_i - \Psi_i < 0$ for $i = 1, ..., m$, go to Step 3.*

Step 3 *Define $k = \max\{i : \bar{a}_i - \Psi_i \geq 0\}$. If $k = 1$, stop; then let $m = k$ and go back to Step 1.*

Proposition 2.3.4 determines the following optimal solution for where it is optimal to purchase from m^* return types, where $1 \leq m^* \leq n$ holds. (The inequality $1 \leq m^*$ holds since $\delta c_0 \geq c_i + \Psi_i$ for all $i = \{1, .., n\}$ holds, i.e., OEM always requires returns from type 1 in the optimal solution.) The global optimal solution is given as follows,

$$
\begin{aligned}
a_j^* &= \frac{\delta c_0 + \delta(1-\delta)\left(\sum_{i=1}^{m^*} \alpha_i (c_i + \Psi_i + \Psi_j - c_j)\right) + \Psi_j - c_j}{2\left(1 + \delta(1-\delta)\sum_{i=1}^{m^*} \alpha_i\right)} \quad \forall j = \{1, .., m^*\}, & (2.12) \\
q_r^* &= \frac{\delta c_0 \sum_{i=1}^{m^*} \alpha_i - \sum_{i=1}^{m^*} \alpha_i (c_i + \Psi_i)}{2\left(1 + \delta(1-\delta)\sum_{i=1}^{m^*} \alpha_i\right)}, & (2.13) \\
q_0^* &= \frac{1 - 2\delta q_r^* - c_0}{2}, & (2.14) \\
p_0^* &= \frac{1 + c_0}{2}, & (2.15) \\
p_r^* &= \delta(p_0^* - (1-\delta) q_r^*). & (2.16)
\end{aligned}
$$

An interesting observation from the optimal solution is that the optimal selling price of the new product, p_0, only depends on the manufacturing cost, c_0, and so remanufacturing does not affect sales price of the new products. This implies that in the optimal solution, the profit (sales price minus cost) per new product is

$$p_0^* - c_0 = \frac{1 - c_0}{2}, \tag{2.17}$$

which again only depends on c_0. Remanufacturing does not affect the profit per new item. This is in line with the results of Atasu et al. (2008). We next consider the profit per a remanufacturing item.

Similarly, from (2.12), (2.13), (2.15), and (2.16) we obtain that

$$p_r^* - (a_i^* + c_i) = \frac{\delta - c_i - \Psi_i}{2}, \tag{2.18}$$

i.e., the profit per remanufactured type i core only depends on the cost structure of that core type and consumers' relative willingness to pay for the remanufactured product.

From (2.18) we also find that the profit earned per type i return does not depend on the cost structure of the other return types. Moreover, the profit per core decreases with the minimum cost $(\Psi_i + c_i)$ of acquiring and remanufacturing a core.

	a_j^*	a_k^*	p_n^*	p_r^*	q_0^*	q_r^*	q_j^*	q_k^*	q_{tot}^*
c_0	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\searrow$
c_j	$\searrow$	$\nearrow$	$\perp$	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$	$\searrow$
Ψ_j	$\nearrow$	$\nearrow$	$\perp$	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$	$\searrow$
δ	$\nearrow$	$\nearrow$	$\perp$	$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow\searrow$

Table 2.2: Effect of parameters on the optimal solution ($\nearrow$: increasing, $\searrow$: decreasing, $\perp$: constant, $\nearrow\searrow$: either increasing or decreasing)

2.4 Sensitivity analyses

In this section, we first analyze the effects of changes on parameters c_0, c_j, Ψ_j, and δ on the optimal solution analytically. Afterwards, we determine more complicated effects of changes α_j on the optimal solution and the effect of all parameters on the profitability of remanufacturing in a numerical study.

Table 2.2 summarizes the effect of c_0, c_j, Ψ_j, and δ on the acquisition price, the selling prices and production quantities. The details of the sensitivity analysis are presented in Appendix 2.C. Instead of repeating all these effects in words, we will describe and explain the most interesting and combined effects.

If the remanufacturing cost of class j, c_j, increases, then the acquisition price offered to that class, a_j^*, decreases; and the number of cores acquired from class j also decreases. However, in order to partially compensate this decrease, as c_j increases, the acquisition prices offered for other return types increase. The total number of remanufactured items decreases with c_j, which leads to an increase in the sales price for the remanufactured products and the amount of manufacturing and a decrease in the total production.

An increase in the minimal acquisition price for type j, Ψ_j, leads to an increase in a_j^* but a decrease in q_j^*. The effects of Ψ_j on the sales prices of the new and remanufactured products and on the production quantities are similar to the effect of c_j.

Finally, Table 2.2 shows that as the consumers willingness to pay for the remanufactured products, δ, increases, remanufactured products are sold at higher prices as expected. Thus, the OEM acquires and remanufactures more cores by setting higher acquisition prices, and manufactures fewer new items. The total production can either increase or decrease with δ.

$Class$	1	2	3	4	5	6
α_i	0.1	0.15	0.2	0.3	0.2	0.3
c_i	0.05	0.15	0.3	0.35	0.4	0.45
Ψ_i	0.17	0.13	0.08	0.06	0.04	0.02

Table 2.3: Parameters used in the numerical study for different classes

For tractability reasons, further sensitivity analysis is conducted in a numerical study to understand the effect of parameters on the optimal pricing and production decisions and also on the profitability of remanufacturing. We consider a base case and study the effect of the parameters in our model by varying one parameter at a time. In the base case, similar to the study in Guide et al. (2003), there are six return types, i.e., $n = 6$. Return coefficients, remanufacturing costs, and minimal acquisition prices for each six classes used in the base case are given in Table 2.3. Note that $c_1 + \Psi_1 \leq ... \leq c_6 + \Psi_6$ holds. For every parameter, we consider two alternative values below and two above the base value. Table 2.4 shows all considered parameter values.

The reasoning behind using these values is as follows. Subramanian and Subramanyam (2012) find that consumers' relative willingness to pay for remanufactured products ranges from 0.60 to 0.85. In line with their findings, in the base case we take $\delta = 0.7$ and consider δ values from 0.60 to 0.85. In this section, we only present the effect of changes in α_2, c_2 and Ψ_2, since changes in α_i, c_i and Ψ_i for $i \neq 2$ will lead to similar insights. Also, we consider type 2 class to understand how a change in the availability of a certain return type affects decisions for more and less preferred return types. To investigate a broad range for the availability of a core type 2, we vary α_2 from 0.01 to 5. We remark that (2.9) and (2.10) hold for all considered parameter settings.

Table 2.5 summarizes the effect of α_2 on the optimal solution. We find that as α_2 increases, implying that it becomes easier to collect cores of type 2, more cores are indeed acquired of this type. However, to compensate, fewer new products and remanufactured products from other core types are produced, and all acquisition prices are reduced.

We next look at the effect of parameters on the OEM's optimal profit and on the relative percentage gain from remanufacturing compared to the non-remanufacturing model. It is easy

c_0	c_2	Ψ_2	α_2	δ
0.55	0.10	0.09	0.01	0.6
0.6	0.12	0.11	0.05	0.65
0.65	**0.15**	**0.13**	**0.15**	**0.7**
0.7	0.17	0.15	0.5	0.75
0.75	0.19	0.17	5	0.85

Table 2.4: Parameters used in the numerical study where base values are indicated in bold

α_2	0.01	0.05	0.15	0.5	5
a_1^*	0.282	0.282	0.280	0.276	0.242
a_2^*	0.212	0.212	0.210	0.206	0.172
a_3^*	0.112	0.112	0.110	0.106	0
a_4^*	0.077	0.077	0.075	0.071	0
a_5^*	0.042	0.042	0.041	0	0
a_6^*	0	0	0	0	0
p_n^*	0.825	0.825	0.825	0.825	0.825
p_r^*	0.573	0.572	0.570	0.566	0.532
q_0^*	0.158	0.156	0.151	0.135	0.023
q_r^*	0.024	0.027	0.034	0.057	0.217
q_{tot}^*	0.182	0.183	0.185	0.192	0.240

Table 2.5: Effect of α_2 on the optimal acquisition and sales prices, and on optimal production quantities

c_0	0.55	0.60	0.65	0.70	0.75
m^*	2	4	5	5	6
% Gain	11.6	23.3	49.5	101.8	212.9
δ	0.60	0.65	0.70	0.75	0.85
m^*	3	4	5	5	6
% Gain	20.2	31.4	49.5	74.4	151.48
c_2	0.10	0.12	0.15	0.17	0.19
m^*	4	4	5	5	5
% Gain	54.1	52.2	49.5	47.7	45.9
Ψ_2	0.09	0.11	0.13	0.15	0.17
m^*	4	5	5	5	5
% Gain	53.2	51.3	49.5	47.7	45.9
α_2	0.01	0.05	0.15	0.50	5
m^*	5	5	5	4	2
% Gain	35.6	39.7	49.5	82.0	290.0

Table 2.6: Effect of parameters on profitability of remanufacturing (The "gain" is the percentage increase in OEM profits from the using remanufacturing option)

to see that without remanufacturing the optimal profit is $\Pi^*_{non} = (1 - c_0)^2/4$. So the relative percentage gain from remanufacturing is given by $(\Pi^* - \Pi^*_{non})/\Pi^*_{non}$.

The effects of parameters on the optimal number of return types and the profitability of remanufacturing are presented in Table 2.6. It appears that as manufacturing becomes more costly, the OEM purchases cores from more return types and the gain from remanufacturing increases. The profit of the OEM, Π^*, may either increase or decrease with c_0. Table 2.6 also shows that as consumers are willing to pay more for the remanufactured products, remanufacturing becomes more profitable and the OEM purchases cores from more return types. As remanufacturing a return from any type becomes more costly (an increase in c_2 or Ψ_2), the gain from remanufacturing decreases and the OEM is more inclined to also purchase cores from less preferred types. Finally, as there are more type-2 returns available, i.e., as α_2 increases, the

OEM acquires more returns from that type and the gain from remanufacturing increases.

2.5 Conclusions

In this chapter, we combine two aspects of remanufacturing activities namely, product acquisition management and pricing of the new and remanufactured products. We consider an OEM that manufactures new items as well as remanufactures returned items. Returned items are classified into different types and the OEM decides both on how much to purchase from each return type (with the acquisition prices offered) and under which price to sell new and remanufactured items, where consumers have lower willingness to pay for the remanufactured items. We derive a procedure to find the closed-form optimal solution. In line with the results of Atasu et al. (2008), we find that the direct profit per a new item only depends on the manufacturing cost. We also find that for the optimal solution, instead of having an equal profit per remanufactured item from all return types, the profit per item is higher for more preferred return types. This is an important observation, as practitioners may find it intuitive and/or fair to apply equal profits, but doing so is suboptimal.

We conclude by pointing out future research directions, related to the limiting assumptions in our model. First of all, we assume a single period setting. Two or multiple period settings can be considered where the number of available returns in a period depends on the product sales in the prior period. Secondly, different selling prices for different core types can be considered. Finally, we inclusion of a competing third party remanufacturer in order to study the impact of competition on firms' optimal behaviors and profit performance can be considered.

Appendix 2.A: Derivation of the Inverse Demand Functions

It is assumed that each consumer uses at most one unit. His/her willingness to pay for that single unit, θ, is distributed uniformly between 0 and 1. Then, the net utility of a consumer, NU, is given by

$$NU = \delta^m \theta - p_z,$$

where $m = 0$ and $z = 0$ if the product is new, and $m = 1$ and $z = r$ if the product is remanufactured. A consumer has three different strategies, based on the value of NU, namely, purchase a new product, strategy N, purchase a remanufactured product, strategy R, and purchase nothing, strategy X. We can derive the inverse demand functions where the market size is normalized to 1. First, we consider the consumer who is indifferent between strategy R and X. In this case, $\theta = 1 - q_0 - q_r$ and this gives

$$NU_R = \delta(1 - q_0 - q_r) - p_r = 0 = NU_X,$$

which implies, $p_r = \delta(1 - q_0 - q_r)$. Then, we consider the consumer who is indifferent between strategy N and R. In this case, $\theta = 1 - q_0$ and this gives

$$NU_N = 1 - q_0 - p_0 = \delta(1 - q_0) - p_r = NU_R,$$

where $p_r = \delta(1 - q_0 - q_r)$. This implies, $p_0 = 1 - q_0 - \delta q_r$.

Appendix 2.B: Proofs of Propositions

Proof of Proposition 2.3.1

To show that the function is concave, we should show that the Hessian is negative definite. The second order conditions are

$$\begin{aligned}
\frac{\partial^2\Pi}{\partial q_0^2} &= -2, & \frac{\partial^2\Pi}{\partial q_0 \partial a_j} = \frac{\partial^2\Pi}{\partial a_j \partial q_0} = -2\delta\alpha_j \\
\frac{\partial^2\Pi}{\partial a_j^2} &= -2\alpha_j(\delta\alpha_j + 1), & \frac{\partial^2\Pi}{\partial a_j \partial a_k} = \frac{\partial^2\Pi}{\partial a_k \partial a_j} = -2\delta\alpha_j\alpha_k
\end{aligned}$$

From these conditions the Hessian matrix is

$$H = \begin{bmatrix} -2 & -2\delta\alpha_1 & -2\delta\alpha_2 & ... & -2\delta\alpha_n \\ -2\delta\alpha_1 & -2\alpha_1(\delta\alpha_1 + 1) & -2\delta\alpha_1\alpha_2 & ... & -2\delta\alpha_1\alpha_n \\ -2\delta\alpha_2 & -2\delta\alpha_1\alpha_2 & -2\alpha_2(\delta\alpha_2 + 1) & ... & -2\delta\alpha_2\alpha_n \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ -2\delta\alpha_n & -2\delta\alpha_1\alpha_n & -2\delta\alpha_2\alpha_n & ... & -2\alpha_n(\delta\alpha_n + 1) \end{bmatrix}.$$

It is known that an $n \times n$ symmetric matrix is negative definite if the determinants of the principal minors have alternating signs starting with a negative sign. Furthermore, elementary row operations will not change the value of a determinant. First, by multiplying row 1 with $-\alpha_j$ and adding to row $1 + j$ for $\forall j \in \{1, .., n\}$ we obtain the following matrix :

$$H' = \begin{bmatrix} -2 & -2\delta\alpha_1 & -2\delta\alpha_2 & ... & -2\delta\alpha_n \\ 2\delta\alpha_1(1-\delta) & -2\alpha_1 & 0 & ... & 0 \\ 2\delta\alpha_2(1-\delta) & 0 & -2\alpha_2 & ... & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 2\delta\alpha_n(1-\delta) & 0 & 0 & ... & -2\alpha_n \end{bmatrix}.$$

Then, by multiplying row $k + 1$ with -2 and adding to row 1 for $\forall k \in \{1, .., n\}$ we obtain the following lower triangular matrix whose principal minors are also lower triangular matrices:

$$H'' = \begin{bmatrix} -2\left(1 + \delta(1-\delta)\sum_{i=1}^{n}\alpha_i\right) & 0 & 0 & ... & 0 \\ 2\delta\alpha_1(1-\delta) & -2\alpha_1 & 0 & ... & 0 \\ 2\delta\alpha_2(1-\delta) & 0 & -2\alpha_2 & ... & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 2\delta\alpha_n(1-\delta) & 0 & 0 & ... & -2\alpha_n \end{bmatrix}.$$

The determinant of a lower triangular matrix is equal to the multiplication of its diagonal elements. Thus, determinants of principal minors will have alternating signs, starting with a negative sign. So we conclude that the Hessian is negative definite and the function is concave.

Proof of Proposition 2.3.2

Since the function is concave, the simultaneous solution of the following first order conditions will give the unique optimal solution for the unconstrained problem.

$$\frac{\partial \Pi}{\partial q_0} = 1 - 2q_n - 2\delta \sum_{i=1}^{n} \alpha_i(a_i - \Psi_i) - c_0 = 0 \tag{2.19}$$

$$\frac{\partial \Pi}{\partial a_j} = \delta\alpha_j \left(1 - 2q_n - 2\sum_{i=1}^{n} \alpha_i(a_i - \Psi_i)\right) - \alpha_j(2a_j - \Psi_j + c_j) = 0 \;\forall j = \{1, .., n\}. \tag{2.20}$$

From equation (2.19) we find that

$$2q_0 = 1 - 2\delta \sum_{i=1}^{n} \alpha_i(a_i - \Psi_i) - c_0. \tag{2.21}$$

Substituting (2.21) into (2.20) we obtain

$$2\delta\alpha_j(\delta - 1)\sum_{i=1}^{n} \alpha_i(a_i - \Psi_i) - \alpha_j(2a_j + c_j - \Psi_j) + \delta\alpha_j c_0 = 0. \tag{2.22}$$

Solving (2.22) for a_j gives

$$a_j = \frac{\delta c_0 - 2\delta(1-\delta)\sum_{i=1}^{k} \alpha_i(a_i - \Psi_i) + \Psi_j - c_j}{2}. \tag{2.23}$$

Using (2.3), we can rewrite (2.23) as

$$a_j = \frac{\delta c_0 - 2\delta(1-\delta)q_r^* + \Psi_j - c_j}{2} \tag{2.24}$$

and get

$$\sum_{j=1}^{n} \alpha_j(a_j^* - \Psi_j) = \sum_{j=1}^{n} \left(\alpha_j \frac{\delta c_0 - 2\delta(1-\delta)q_r^* - \Psi_j - c_j}{2} \right) = q_r^*. \tag{2.25}$$

From (2.25), we then obtain

$$q_r^* = \frac{\delta c_0 \sum_{i=1}^{n} \alpha_i - \sum_{i=1}^{n} \alpha_i(c_i + \Psi_i)}{2\left(1 + \delta(1-\delta)\sum_{i=1}^{n} \alpha_i\right)}. \tag{2.26}$$

Substituting (2.26) into (2.24) gives a_j^*. The value for p_0^* can be found by substituting (2.21) in to (2.1).

Proof of Lemma 2.3.1

For $1 \leq i < k < n$, combining

$$a_i - \Psi_i = \frac{\delta c_0 + \delta(1-\delta)\left(\sum_{\substack{j=1 \\ j \neq i}}^{n} \alpha_j(c_j + \Psi_j - \Psi_i - c_i)\right) - \Psi_i - c_i}{2\left(1 + \delta(1-\delta)\sum_{j=1}^{n} \alpha_j\right)}$$

and

$$a_k - \Psi_k = \frac{\delta c_0 + \delta(1-\delta)\left(\sum_{\substack{j=1 \\ j \neq k}}^{n} \alpha_j(c_j + \Psi_j - \Psi_k - c_k)\right) - \Psi_k - c_k}{2\left(1 + \delta(1-\delta)\sum_{j=1}^{n} \alpha_j\right)}$$

we get

$$\begin{aligned}(a_k - \Psi_k) - (a_i - \Psi_i) \;=\; & \frac{\delta c_0 + \delta(1-\delta)\left(\sum_{\substack{j=1 \\ j \neq k}}^{n} \alpha_j(c_j + \Psi_j - \Psi_k - c_k)\right) - \Psi_k - c_k}{2\left(1 + \delta(1-\delta)\sum_{j=1}^{n} \alpha_j\right)} \\ & - \frac{\delta c_0 + \delta(1-\delta)\left(\sum_{\substack{j=1 \\ j \neq i}}^{n} \alpha_j(c_j + \Psi_j - \Psi_i - c_i)\right) - \Psi_i - c_i}{2\left(1 + \delta(1-\delta)\sum_{j=1}^{n} \alpha_j\right)}\end{aligned}$$

$$= \frac{\delta(1-\delta)\left(\sum_{\substack{j=1\\ j\neq k\\ j\neq i}}^{n}\alpha_j(c_j+\Psi_j) - \sum_{\substack{j=1\\ j\neq k\\ j\neq i}}^{n}\alpha_j(\Psi_k+c_k) + \alpha_i(c_i+\Psi_i) - \alpha_i(\Psi_k+c_k)\right) - \Psi_k - c_k}{2\left(1+\delta(1-\delta)\sum_{j=1}^{n}\alpha_j\right)}$$

$$- \frac{\delta(1-\delta)\left(\sum_{\substack{j=1\\ j\neq k\\ j\neq i}}^{n}\alpha_j(c_j+\Psi_j) - \sum_{\substack{j=1\\ j\neq k\\ j\neq i}}^{n}\alpha_j(\Psi_i+c_i) + \alpha_k(c_k+\Psi_k) - \alpha_k(\Psi_i+c_i)\right) - \Psi_i - c_i}{2\left(1+\delta(1-\delta)\sum_{j=1}^{n}\alpha_j\right)}$$

$$= \frac{\delta(1-\delta)\left((c_i+\Psi_i-\Psi_k-c_k)\sum_{\substack{j=1\\ j\neq k\\ j\neq i}}^{n}\alpha_j + (\alpha_i+\alpha_k)(c_i+\Psi_i-\Psi_k-c_k)\right) - \Psi_k - c_k + \Psi_i + c_i}{2\left(1+\delta(1-\delta)\sum_{j=1}^{n}\alpha_j\right)}$$

$$= \frac{\delta(1-\delta)(c_i+\Psi_i-\Psi_k-c_k)\left(\sum_{j=1}^{n}\alpha_j\right) - \Psi_k - c_k + \Psi_i + c_i}{2\left(1+\delta(1-\delta)\sum_{j=1}^{n}\alpha_j\right)} = \frac{-\Psi_k - c_k + \Psi_i + c_i}{2} \leq 0, \tag{2.27}$$

where the last inequality comes from the assumption that $c_i + \Psi_i \leq c_k + \Psi_k$. Thus, it can never be optimal to buy from class k but not from some class $i < k$.

Proof of Lemma 2.3.2

Using (2.5) we get

$$\begin{aligned}
\overline{a}_j^{m-1} - \overline{a}_j^{m} &= \frac{\delta c_0 + \delta(1-\delta)\left(\sum_{i=1}^{m-1}\alpha_i(c_i+\Psi_i+\Psi_j-c_j)\right) + \Psi_j - c_j}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m-1}\alpha_i\right)} \\
&\quad - \frac{\delta c_0 + \delta(1-\delta)\left(\sum_{i=1}^{m}\alpha_i(c_i+\Psi_i+\Psi_j-c_j)\right) + \Psi_j - c_j}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m}\alpha_i\right)} \\
&= \frac{\delta(1-\delta)\alpha_m\left(\delta c_0 + \delta(1-\delta)\left(\sum_{j=1}^{m-1}\alpha_j(c_j+\Psi_j-\Psi_m-c_m)\right) - \Psi_m - c_m\right)}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m-1}\alpha_i\right)\left(1+\delta(1-\delta)\sum_{i=1}^{m}\alpha_i\right)} \\
&= \frac{\delta(1-\delta)\alpha_m\left(\overline{a}_m^{m} - \Psi_m\right)}{\left(1+\delta(1-\delta)\sum_{i=1}^{m-1}\alpha_i\right)}.
\end{aligned}$$

Appendix 2.C: Sensitivity analyses

The optimal solution for a given number of classes are given in Proposition 2.3.4. In the following sections, we present the details of the sensitivity analyses for c_0, c_i, Ψ_i, and δ.

Sensitivity analysis w.r.t c_0

$$\begin{aligned}
\frac{\partial a_j^*}{\partial c_0} &= \frac{\partial a_k^*}{\partial c_0} = \frac{\delta}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} > 0,\\
\frac{\partial q_j^*}{\partial c_0} &= \frac{\alpha_j \delta}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} > 0,\\
\frac{\partial q_k^*}{\partial c_0} &= \frac{\alpha_k \delta}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} > 0,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial p_0^*}{\partial c_0} &= \frac{1}{2} > 0,\\
\frac{\partial p_r^*}{\partial c_0} &= \delta\left(\frac{\partial p_n^*}{\partial c_n} - (1-\delta)\frac{\partial q_r^*}{\partial c_n}\right) = \frac{\delta}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} > 0,\\
\frac{\partial q_0^*}{\partial c_0} &= -\delta\frac{\partial q_r^*}{\partial c_0} - \frac{1}{2} < 0,\\
\frac{\partial q_r^*}{\partial c_0} &= \frac{\delta\sum_{i=1}^{m^*}\alpha_i}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} > 0,\\
\frac{\partial q_{tot}^*}{\partial c_0} &= (1-\delta)\frac{\partial q_r^*}{\partial c_0} - \frac{1}{2} = \frac{(1-\delta)\delta\sum_{i=1}^{m^*}\alpha_i}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} - \frac{1}{2}\\
&= -\frac{1}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} < 0.
\end{aligned}$$

Therefore, as c_0 increases, a_j^*, p_r^*, p_0^*, and q_r^* increase and q_0^* and q_{tot}^* decrease.

Sensitivity analysis w.r.t c_j

$$\begin{aligned}
\frac{\partial a_j^*}{\partial c_j} &= \frac{\partial q_j^*}{\partial c_j} = \frac{-1-\delta(1-\delta)\sum_{\substack{i=1\\i\neq j}}^{m^*}\alpha_i}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} < 0,\\
\frac{\partial a_j^*}{\partial c_k} &= \frac{\delta(1-\delta)\sum_{\substack{i=1\\i\neq j}}^{m^*}\alpha_i}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} > 0,\ k\neq j\\
\frac{\partial p_0^*}{\partial c_j} &= 0,\\
\frac{\partial p_r^*}{\partial c_j} &= -\delta(1-\delta)\frac{\partial q_r^*}{\partial c_j} = \frac{\delta(1-\delta)\alpha_j}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} > 0,\\
\frac{\partial q_0^*}{\partial c_j} &= -\delta\frac{\partial q_r^*}{\partial c_j} = \frac{\delta\alpha_j}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} > 0,\\
\frac{\partial q_r^*}{\partial c_j} &= \frac{-\alpha_j}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} < 0,\\
\frac{\partial q_{tot}^*}{\partial c_j} &= (1-\delta)\frac{\partial q_r^*}{\partial c_j} = -\frac{(1-\delta)\alpha_j}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} < 0.
\end{aligned}$$

Therefore, as c_j increases, a_j^* and q_r^* decrease and p_r^*, q_0^*, and q_{tot}^* increase, and p_0^* does not change. Also, as c_k increases, a_j^* increases.

Sensitivity analysis w.r.t Ψ_j

$$\frac{\partial a_j^*}{\partial \Psi_j} = \frac{\delta(1-\delta)\left(2\alpha_j+\sum_{\substack{i=1\\i\neq j}}^{m^*}\alpha_i\right)}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} < 0,$$

$$
\begin{aligned}
\frac{\partial q_j^*}{\partial \Psi_j} &= \frac{-1-\delta(1-\delta)\sum_{\substack{i=1\\ i\neq j}}^{m^*}\alpha_i}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} < 0,\\
\frac{\partial a_j^*}{\partial \Psi_k} &= \frac{\partial q_j^*}{\partial \Psi_k} = \frac{\delta(1-\delta)\sum_{\substack{i=1\\ i\neq j}}^{m^*}\alpha_i}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} > 0 \; k \neq j,\\
\frac{\partial p_0^*}{\partial \Psi_j} &= 0,\\
\frac{\partial p_r^*}{\partial \Psi_j} &= -\delta(1-\delta)\frac{\partial q_r^*}{\partial c_j} = \frac{\delta(1-\delta)\alpha_j}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} > 0,\\
\frac{\partial q_0^*}{\partial \Psi_j} &= -\delta\frac{\partial q_r^*}{\partial c_j} = \frac{\delta\alpha_j}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} > 0,\\
\frac{\partial q_r^*}{\partial \Psi_j} &= \frac{-\alpha_j}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} < 0,\\
\frac{\partial q_{tot}^*}{\partial \Psi_j} &= (1-\delta)\frac{\partial q_r^*}{\partial c_j} = -\frac{(1-\delta)\alpha_j}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)} < 0.
\end{aligned}
$$

Therefore, as Ψ_j increases, q_r^* decreases and a_j^*, p_r^*, q_0^*, and q_{tot}^* increase, and p_0^* does not change. Also, as Ψ_k increases, a_j^* increases.

Sensitivity analysis w.r.t δ

It is obvious that p_0^* does not change with δ.

$$
\begin{aligned}
\frac{\partial a_j^*}{\partial \delta} &= \frac{\partial a_k^*}{\partial \delta} = \frac{c_0(1+\delta^2\sum_{i=1}^{m^*}\alpha_i + (1-2\delta)(\sum_{i=1}^{m^*}\alpha_i(c_i+\Psi_i))}{2\left(1+\delta(1-\delta)\sum_{i=1}\alpha_i\right)^2} > 0,\\
\frac{\partial q_{\{j\}}^*}{\partial \delta} &= \alpha_j\frac{\partial a_j^*}{\partial \delta} > 0, \frac{\partial q_k^*}{\partial \delta} = \alpha_k\frac{\partial a_k^*}{\partial \delta} > 0,\\
\frac{\partial q_r^*}{\partial \delta} &= \frac{\sum_{i=1}^{m^*}\alpha_i\left(c_0+\delta\left(\sum_{i=1}^{m^*}\alpha_i(\delta c_0 - c_i - \Psi_i)\right) + \sum_{i=1}^{m^*}\alpha_i(c_i+\Psi_i)(1-\delta)\right)}{2\left(1+\delta(1-\delta)\sum_{i=1}^{m^*}\alpha_i\right)^2} > 0.
\end{aligned}
$$

From (2.9) it is known that $\sum_{i=1}^{m^*}\alpha_i(\delta c_0 - c_i - \Psi_i) \geq 0$ holds. Thus, q_r increases as δ increases.

$$
\frac{\partial q_0^*}{\partial \delta} = -1\left(q_r + \delta\frac{\partial q_r^*}{\partial \delta}\right) < 0.
$$

Thus, as δ increases, q_0 decreases.

Let $\sum_{i=1}^{m^*}\alpha_i = A$ and $\sum_{i=1}^{m^*}\alpha_i(c_i+\Psi_i) = B$. Then, we get

$$
\begin{aligned}
\frac{\partial p_r^*}{\partial \delta} &= \frac{c_0 + B - 2\delta^2 A - 2\delta B + \delta^2 A^2 - 2\delta^3 A^2 + \delta^4 A^2 + 1 + 2\delta A + \delta^2 c_0 A}{2(1+\delta A - \delta^2 A)^2}\\
&= \frac{\delta A(2(1-\delta) + \delta A(1-\delta)^2) + \delta(\delta c_0 A - B) + 1 + c_0 + B(1-\delta)}{2(1+\delta A - \delta^2 A)^2} > 0,
\end{aligned}
$$

where the last inequality results from (2.9). Thus, as δ increases, p_r increases.

$$\begin{aligned}
\frac{\partial q_{tot}^*}{\partial \delta} &= (1-\delta)\frac{\partial q_r^*}{\partial \delta} - q_r^* \\
&= \frac{Ac_0(1-\delta) + AB(1-\delta)^2 - (\delta c_0 A - B)}{2(1+\delta A - \delta^2 A)^2}
\end{aligned}$$

which can be both negative and positive.

Chapter 3

Capacity and production decisions under a remanufacturing strategy

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In this chapter, we investigate the effect of remanufacturing on capacity and production decisions. Inspired by the situation for a specific car company, we analyze a two-period model with manufacturing in both periods and the option in the second period to remanufacture products that are returned/collected at the end of the first period. We first and foremost focus on the case where remanufacturing is less costly and less capacity intensive than manufacturing. This setting is realistic and obviously the one where remanufacturing is most beneficial. Optimal manufacturing and remanufacturing quantities are derived and it is analyzed under what conditions (specified by costs, capacity restrictions and demand) remanufacturing leads to increased total production. We also consider the cases where remanufacturing is either more costly or more capacity intensive than manufacturing, and contrast the results the those of our main case. One particularly insightful find is that remanufacturing is seldom (very) profitable if it is more costly than manufacturing, and hence that companies should focus their attention on situations where remanufacturing lowers costs.

3.1 Introduction

Over the last 50 years, the level of consumption by growing population has been continuously increasing. As a result, the world now faces serious environmental problems such as waste with presence of toxic materials and depletion of natural resources (Pochampally et al., 2009). Driven by legislation and societal pressure to mitigate these environmental problems, and also by economic incentives, more and more firms are starting remanufacturing operations next to the traditional manufacturing operations (Tang and Teunter, 2006).

Remanufacturing is the process of bringing used products to a “like-new” functional state with warranty to match. It has numerous benefits for original equipment manufacturers (OEMs), such as savings in labor, material and energy costs. By adopting remanufacturing firms can save between 40% and 60% of the cost of manufacturing a new product while using only 20% of the energy (Guide et al., 1997). In the 2008/2009 financial year, Fuji Xerox Australia remanufactured more than 230,000 equipment parts, equating to a $6 million cost-saving compared to sourcing new parts. Furthermore, remanufacturing leads to shorter production lead times; balanced production lines; new market development opportunities, and a positive, socially concerned image for firms (McConocha and Speh, 1991). Caterpillar created a new market among contractors who cannot afford to buy a Caterpillar product outright by adopting remanufacturing as a part of production strategy (Gutowski et al., 2001). In addition to these benefits, remanufacturing may offer a better alternative to capacity constraint on new product manufacturing (Atasu et al., 2008).

In this chapter, we consider remanufacturing in a two-period, capacitated production setting and our aim is to determine the effect of remanufacturing on capacity and production. More specifically, we address the following questions.

1) Under what conditions is remanufacturing profitable? If remanufacturing is either more costly or more capacity intensive, can it still be profitable?

2) What is the impact of remanufacturing on the optimal capacity and production decisions?

3) How will market conditions and cost structures effect the profitability of remanufacturing?

We construct a model to optimize capacity, manufacturing and remanufacturing decisions. The model is motivated by a specific case company which manufactures and remanufactures car parts (Tang and Teunter, 2006). The details of the case and mathematical model are described in the next section.

Capacity management has been widely studied in the supply chain literature. The three main areas that have been addressed are production, inventory and demand management; real options; and risk sharing and vertical integration between suppliers and buyers through capacity reservation contracts. Wu et al. (2005) provide an extensive review on capacity expansion tactics in the high-tech industry.

There are numerous studies on closed-loop supply chains and remanufacturing in the current literature. Fleischmann et al. (1997) provide an excellent review and Guide and Van Wassenhove (2009) describe the evolution of the research on closed- loop supply chains. A recent survey on production planning and control for remanufacturing is provided by Junior and Filho (2012). In the remanufacturing literature, there are relatively few studies that consider capacitated settings. In this literature stream some studies only focus on the planning remanufacturing activities capacities without considering production capacity of new products. For instance, Guide et al. (1997) consider remanufacturing capacity by taking into account material recovery rates and stochastic routings, and they evaluate the performance of several capacity planning techniques. Aksoy and Gupta (2001) analyze the trade-off between increasing the number of buffers and increasing the capacity at the remanufacturing stations with uncertainties in the operational environment. They use an open queuing network to model the remanufacturing system. Franke et al. (2006) consider remanufacturing capacity for the mobile phone industry. They introduce a linear programming model for the planning of remanufacturing capacities and production programs. Georgiadis et al. (2006) analyze capacity expansion/contraction of collection and remanufacturing activities considering product lifecyle and return patterns. They adopt system dynamics methodology to derive dynamic capacity planning policies. Another study that uses system dynamics methodology is conducted by Vlachos et al. (2007). They study the long-term behavior of reverse supply chains with remanufacturing, and propose efficient remanufacturing and collection expansion policies. They also include specific external factors such as obligations and penalties imposed by legislation that influence profits, costs and flows. Different from these studies, we consider production capacity for both new and remanufactured products.

Other studies that consider capacitated production setting for both new and remanufactured products do exist. Debo et al. (2006) analyze the introduction and management of remanufactured products considering life-cycles of products. They also focus on capacitated settings and try to understand the impact of the product diffusion rate on the capacity requirements for new and remanufactured products. Additionally, they investigate the relative value of flexible capacity which can be used to both manufacture and remanufacture products, compared to dedicated capacity for each activity. Bayindir et al. (2003) investigate the conditions on different system parameters, including capacity of the production facility, for which the remanufacturing option provides cost benefits. They model the production environment as a queuing network, where manufacturing and remanufacturing require both common and separated operations. They also assume that there is no difference between remanufactured and manufactured products. Bayindir et al. (2007) relax this assumption and investigate the profitability of having a remanufacturing option when the manufactured and remanufactured products are segmented to different markets and production capacity is finite. They consider a single period profit model where the retail price of the new and remanufactured products are fixed. Rubio and Corominas (2008) consider a lean production environment with known and constant demand, and propose a model where manufacturing and remanufacturing capacities can be adjusted. Different from these studies, we investigate the effect of remanufacturing on capacity and production (pricing) decisions in a two-period setting that consists of a growth phase and a maturity phase for a product. Two-period models have been used in the remanufacturing literature in several studies including these, by Majumder and Groenevelt (2001), Ferguson and Toktay (2006), Ferrer and Swaminathan (2006), and Webster and Mitra (2007). In these studies, it is assumed that there is infinite production capacity of new and remanufactured products whereas in this chapter, we consider a capacitated setting.

The rest of the chapter is organized as follows. The next section introduces the motivating case and describes the corresponding model in detail. We characterize the optimal policy for the case in which remanufacturing is less costly as well as less capacity intensive compared to manufacturing in Section 3.3. We further conduct a sensitivity analysis on the optimal solution in Section 3.4 to understand the effect of each parameter on the optimal solution. Also, by comparing to the case where the OEM only manufactures, we gain insights into the effect of remanufacturing on total production, capacity investment and retail prices. In Section 3.5, we relax the assumption that remanufacturing reduces both cost and capacity requirements. We again characterize optimal policies and also study numerically whether remanufacturing can still be profitable in such cases in Section 3.6. Section 3.7 ends with a brief summary of the findings, managerial insights, and avenues for further research.

3.2 Model

In this section, we first introduce the details of the motivating case then construct the mathematical model with related assumptions.

The motivating case for this chapter is a specific car company whose major products are diesel engines, petrol engines, water pumps, cylinder heads, crankshafts, and short blocks. For this study, we focus on a specific product, the water pumps for diesel engines. The remanufacturing processes are very similar to those for manufacturing except for the source of the materials, therefore both manufacturing and remanufacturing are performed on the same production line.

In the mathematical model we assume that the product life-cycle is split into two periods which we can interpret in the following way. In the first period (growth phase), the OEM builds its production capacity and introduces the new product to the market. The number of manufactured new products, q_{1n}, in that period is, of course, restricted by the production capacity Q, i.e., $q_{1n} \leq Q$. In the second period (maturity phase) the product is already in the market and sales continue. Also, the returns from the first period's sales (where γ denotes the fraction that are returned) are received. In the second period, capacity is fixed (from the first period). We do allow the second period to have a different length than the first. Letting θ denote its relative length compared to that of the first period, this gives a capacity of manufacturing θQ new products in the second period. However, an OEM can use part of that capacity to remanufacture used products that are returned/collected at the end of period 1.

The relative capacity requirement for remanufacturing (per remanufactured product) is denoted by τ . So, letting q_{2n} and q_{2r} denote the manufactured and remanufactured products in period 2, we get the following capacity restriction for period 2:

$$q_{2n} + \tau q_{2r} \leq \theta Q.$$

Remanufactured water pumps are sold with the same quality and warranty as the new product and OEM offers a (fixed) discount per remanufactured product. Newly manufactured and remanufactured water pump are used to fulfill the demand for spare parts for engines and demand for new engine assembly. The buyers are assumed to be indifferent between purchasing a newly manufactured product and a remanufactured product that is sold at a fixed discount. To avoid unnecessary notation, we include that "discount" for remanufactured products in our model by adding it to the remanufacturing cost, c_r and use the same selling price for new and remanufactured products.

The relevant costs are those for building capacity (c_o per unit of capacity), for manufacturing (c_n per product) and for remanufacturing (c_r per product). We remark that the model and analysis can easily be generalized from a linear capacity cost function to a general convex function. Additionally, c_r is composed of material cost, acquisition cost for cores and the discount as explained. The market share in both periods is based on to the following linear demand model, where M_i is the market size, a_i is the price sensitivity factor and p_i the selling price, in period i.

$$q_i = M_i - a_i p_i, \; i = 1, 2.$$

or equivalently,

$$p_i = (M_i - q_i)/a_i, \; i = 1, 2.$$

To ensure nonnegative selling prices both in period 1 and 2, we require $q_1 \leq M_1$ and $q_2 \leq M_2$.

Obviously, remanufacturing is particularly interesting for OEMs if it is less costly and less capital-intensive than manufacturing. For this reason, we first assume that $c_n > c_r$ and $\tau < 1$ and find the optimal solution in the next section. In Section 3.5, we relax this assumption and also analyze the cases where remanufacturing is either more costly or more capacity intensive than manufacturing.

To simplify the presentation of the analysis and results, we assume in what remains that the capacity in period 2 is sufficient for remanufacturing all available returns (and manufacture some new items), i.e., $\theta \geq \tau\gamma$. This is realistic as both τ and γ are typically considerably smaller than 1, and the maturity phase not (very) short compared to the growth phase.

The objective is to maximize the total profit over both periods, where revenues and costs in the second period are discounted with factor β. In mathematical terms, we have

$$\begin{aligned} \max \Pi_2(p_2, q_{2n}, q_{2r}) &= p_2 q_2 - c_n q_{2n} - c_r q_{2r}, \qquad (3.1) \\ q_2 &= q_{2n} + q_{2r}, \\ q_{2n} + \tau q_{2r} &\leq \theta Q, \\ q_{2r} &\leq \gamma q_{1n}, \\ q_{2n} &\geq 0, \ q_{2r} \geq 0. \end{aligned}$$

for period 2, and

$$\begin{aligned} \Pi_1(q_{1n}, Q) &= (p_1 - c_n) q_{1n} - c_o Q + \beta \Pi_2^*(q_{1n}, Q), \qquad (3.2) \\ q_{1n} &\leq Q, \\ q_{1n} &\geq 0, \ Q \geq 0 \end{aligned}$$

for period 1, where $\Pi_2^*(q_{1n}, Q)$ denotes the maximum profit in period 2 given q_{1n} and Q.

A list of notations is given in Table 3.1. In addition to parameters and variables introduced previously, some extra expressions that will turn out to be useful for the analysis are included. The marginal production cost in period 1 if there is sufficient capacity in period 2, including future cost savings of remanufacturing $\beta\gamma(c_n - c_r)$, is denoted by $\Delta_0 = c_n + c_o - \beta\gamma(c_n - c_r)$. If there is insufficient capacity in period 2, the marginal benefit of capacity increase in period 1 is $c_o[\theta + (1-\tau)\gamma]/\theta$. Since the marginal cost in period 1 is c_o, the unit capacity cost saving of remanufacturing is

$$c_o[\theta + (1-\tau)\gamma]/\theta - c_o = c_o(1-\tau)\gamma/\theta.$$

Thus, $\Delta_1 = c_n - c_o\gamma(1-\tau)/\theta - \beta\gamma(c_n - c_r)$ is the marginal production cost in period 1 if there is insufficient capacity in period 2. Moreover, Ψ_1 is the desirable production quantity (including both new and remanufacturing quantities) and Ψ_2 is the desirable remanufacturing quantity in period 2 without capacity constraint. Further explanation of Ψ_1 and Ψ_2 will be provided in the next section.

θ	Relative length of the second period compared to the first
τ	Relative capacity requirement of remanufacturing
$M_i,\ i = 1, 2$	Market size in period i
$p_i,\ i = 1, 2$	Selling price in period i
$q_{in},\ i = 1, 2$	Number of new products manufactured in period i
q_{2r}	Number of used products remanufactured in period 2
$q_i,\ i = 1, 2$	Total number of products produced in period i ($q_1 = q_{1n}, q_2 = q_{2n} + q_{2r}$)
Q	Production capacity (in period 1)
c_n	Manufacturing cost (per unit)
c_r	Remanufacturing cost (per unit)
c_o	Capacity cost (per unit)
γ	Return rate
$a_i,\ i = 1, 2$	Price sensitivity factor in period i
β	Discount factor for period 2
Δ_0	$c_n + c_o - \beta\gamma(c_n - c_r)$
Δ_1	$c_n - c_o\frac{(1-\tau)\gamma}{\theta} - \beta\gamma(c_n - c_r)$
Ψ_0	$\frac{M_2 - a_2 c_n - \frac{a_2(c_r - c_n)}{(1-\tau)}}{2}$
Ψ_1	$\frac{M_2 - a_2 c_n}{2}$
Ψ_2	$\frac{M_2 - a_2 c_r}{2}$

Table 3.1: Notations

3.3 Optimal policy if remanufacturing is less costly and less capacity intensive

In the main text, we discuss the most realistic case where (a) manufacturing in period 2 can be profitable, i.e., $M_2 > a_2 c_n$; and (b) manufacturing in period 1 can be profitable, i.e., $M_1 > a_1 \Delta_0$. The details of the characterization of the optimal policy under these conditions is given in Appendix 3.A, and the characterization of the optimal policy if either of these assumptions does not hold given in Appendix 3.C.

The solution procedure is as follows. We start by finding the optimal manufacturing and remanufacturing quantities in period 2 given values for the capacity and the manufacturing quantity in period 1. It will appear that multiple cases have to be considered, depending on whether or not capacity and non negativity constraints are binding. Before considering the different cases, we start with the following proposition regarding to OEM's preference for remanufacturing over manufacturing in period 2.

Proposition 3.3.1 *In period 2, the OEM does not manufacture new products as long as cores are available for remanufacturing.*

In our setting (see Section 3.2) where remanufacturing is less expensive than manufacturing (i.e., $c_r < c_n$) and requires less capacity (as $\tau < 1$), this proposition obviously holds. It also follows directly from the profit function in (3.1).

If we ignore the constraints in (3.1), then it is easy to see that the optimal remanufacturing quantity is $\Psi_2 = (M_2 - a_2 c_r)/2$. Combined with Proposition 3.3.1, this implies that it is optimal to remanufacture Ψ_2 returns when available and all returns otherwise. Recall from Section 3.2 that the capacity is always sufficient to remanufacture all returns (and manufacture some new items). Furthermore, it also directly follows from (3.1) that if the number of returns (γq_{1n}) is less than $\Psi_1 = (M_2 - a_2 c_n)/2$, then it is optimal to remanufacture all returns and additionally manufacture $\Psi_1 - \gamma q_{1n}$ new items, or if there is insufficient capacity, manufacture as many as possible.

Using Ψ_2 and Ψ_1, we next distinguish multiple cases that differ in production type and capacity availability. If $\gamma q_{1n} < \Psi_1$, then all returns are remanufactured and some additional

	Capacity availability		(Re)manufacturing
Case	Period 1	Period 2	Period 2
1	Surplus ($q_{1n} < Q$)	Shortage ($(1-\tau)\gamma q_{1n} + \theta Q < \Psi_1$)	All returns, some new ($\gamma q_{1n} < \Psi_1$)
2	Match ($q_{1n} = Q$)	Shortage ($(1-\tau)\gamma q_{1n} + \theta Q < \Psi_1$)	All returns, some new ($\gamma q_{1n} < \Psi_1$)
3	Surplus ($q_{1n} < Q$)	Match ($(1-\tau)\gamma q_{1n} + \theta Q = \Psi_1$)	All returns, some new ($\gamma q_{1n} < \Psi_1$)
4	Match ($q_{1n} = Q$)	Match ($(1-\tau)\gamma q_{1n} + \theta Q = \Psi_1$)	All returns, some new ($\gamma q_{1n} < \Psi_1$)
5	Match ($q_{1n} = Q$)	Surplus ($(1-\tau)\gamma q_{1n} + \theta Q > \Psi_1$)	All returns, some new ($\gamma q_{1n} < \Psi_1$)
6	Match ($q_{1n} = Q$)	Surplus ($(1-\tau)\gamma q_{1n} + \theta Q > \gamma q_{1n}$)	All returns, no new ($\gamma q_{1n} = \Psi_1$)
7	Match ($q_{1n} = Q$)	Surplus ($(1-\tau)\gamma q_{1n} + \theta Q > \gamma q_{1n}$)	All returns, no new ($\Psi_1 < \gamma q_{1n} \leq \Psi_2$)
8	Match ($q_{1n} = Q$)	Surplus ($(1-\tau)\gamma q_{1n} + \theta Q > \gamma q_{1n}$)	Some returns, no new ($\gamma q_{1n} > \Psi_2$)

Table 3.2: Description of policies for eight cases

new items (up to Ψ_1, depending on capacity) are manufactured in period 2. This gives three cases depending on whether there is a capacity shortage, match or surplus. The first two cases are each split into two subcases depending on whether there is a capacity match ($q_{1n} = Q$) or surplus ($q_{1n} < Q$) in period 1. Obviously, there cannot be a capacity shortage in period 1. Also, it cannot be optimal to have a capacity surplus in both periods, and therefore there has to be a capacity match in period 1 for the case with a capacity surplus in period 2. This leads to five cases for $\gamma q_{1n} < \Psi_1$ that are numbered 1-5 in Table 3.2.

If $\Psi_1 \leq \gamma q_{1n} \leq \Psi_2$, then all returns are remanufactured and no new items are manufactured. It will appear that the analysis of this situation is different for $\gamma q_{1n} = \Psi_1$ and $\Psi_1 < \gamma q_{1n} \leq \Psi_2$, and therefore we consider these as separate cases, numbered 6 and 7, respectively. Finally (case 8), if $\gamma q_{1n} > \Psi_2$ then some but not all returns are remanufactured. For cases 6-8, there is a capacity surplus in period 2 (as the capacity is always sufficient to remanufacture all returns and manufacture some new items - see Section 3.2), and therefore there has to be a capacity match in period 1.

In order to find the global optimal solution, we must find the local optimal solution for every separate case and compare the associated profits. The analysis is straightforward but tedious, and given in Appendix 3.A. It turns out that all cases except 3 can be optimal, and that the optimality conditions are as given in Table 3.3.

Case	Optimality Condition
1	$M_2 - a_2c_n > [\theta + (1-\tau)\gamma](M_1 - a_1\Delta_1) + \frac{a_2c_o}{\beta\theta}$
2	$[(1-\tau)\gamma + \theta](M_1 - a_1\Delta_0) < M_2 - a_2c_n \leq [\theta + (1-\tau)\gamma](M_1 - a_1\Delta_1) + \frac{a_2c_o}{\beta\theta}$
4	$M_2 - a_2c_n = \gamma(M_1 - a_1\Delta_0)$
5	$\gamma(M_1 - a_1\Delta_0) < M_2 - a_2c_n < [(1-\tau)\gamma + \theta](M_1 - a_1\Delta_0)$
6	$M_2 - a_2c_n = [(1-\tau)\gamma + \theta](M_1 - a_1\Delta_0)$
7	$M_2 - a_2c_n < \gamma(M_1 - a_1\Delta_0), M_2 - a_2c_r \geq \gamma[M_1 - a_1(c_n + c_o)]$
8	$M_2 - a_2c_r < \gamma[M_1 - a_1(c_n + c_o)]$

Table 3.3: Optimality conditions for all cases

Note from Table 3.3 that the optimality conditions are all linear in the market sizes of both periods, which leads us to the graphical presentation of the optimal market sizes in Figure 3.1. This figure contains five Regions corresponding to cases 1, 2, 5, 7 and 8, which we will refer to as Regions I-V in this order. As can been seen from Table 3.3, cases 4 and 6 correspond to a line rather than a region in Figure 3.1. Indeed, case 4 is optimal on the boundary between Regions II and III, and case 6 is optimal on the boundary between Regions III and IV. Note

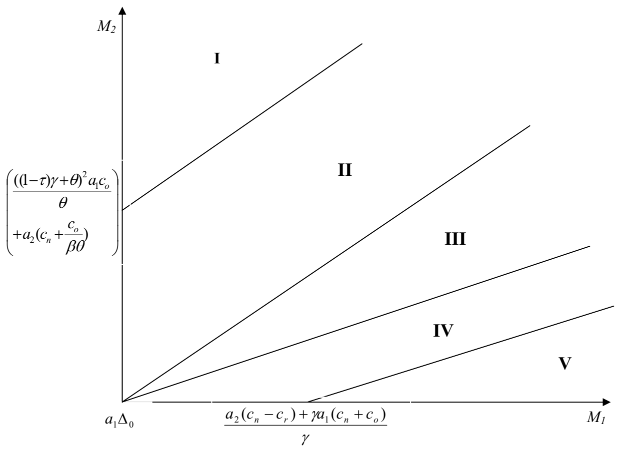

Figure 3.1: Optimality Regions

from Figure 3.1 that for any large enough value of the market size in period 1, the optimal solution starts in Region V for very small market sizes M_2 in period 2, and moves to Regions IV, III, II and I, in this order, as M_2 increases. This is explained as follows. If the market size in period 2 is smaller than that in period 1, there is a sufficient supply of returns to be remanufactured so that manufacturing is not needed. Indeed, for a very small market size only part of the returns is remanufactured (Region V), but for M_2 large enough it becomes profitable to remanufacture all returns (Region IV) as the sales price increases with the market size. Since manufacturing is more costly than remanufacturing, a further increase in M_2 is needed before manufacturing additional new items becomes profitable. If M_2 is just above the threshold where manufacturing becomes profitable, then the number of manufactured items is small and hence there is a capacity surplus (Region III). As M_2 continues to increase, it becomes profitable to use all available production capacity, and a capacity surplus changes into a shortage as more items can be manufactured profitably than capacity allows. At first, the lost profit associated with a capacity shortage in period 2 does not weigh up against the cost of building additional capacity in period 1, and hence there is still a capacity match in that period (Region II). However, as M_2 and with it the lost profit increases, it does become profitable at some point to have a capacity surplus in period 1 (Region I).

Table 3.4 gives the global optimal solution for all regions. We remark that (see Appendix 3.A) cases 4-6 have the same solution and these are therefore included in Region III of this table. Correspondingly, the optimality conditions for cases 4-6 in Table 3.3 can be combined into a single optimality condition.

3.4 Sensitivity analysis

In Table 3.5, the results of the sensitivity analysis are presented. The details of the sensitivity study are provided in Appendix 3.D. Note that these results are complete, but for a few cells corresponding to case 2. For this case, we have not been able to prove what type of effect a change in θ and τ has on the production quantities in period 2, most likely because those effects have a more complex nature. In what follows, rather than present a repetitive discussion of all individual effects, we discuss the most valuable insights.

We first discuss the effect of a change in the remanufacturing cost c_r, which is obviously a key

Region	Cases	Period 1	Period 2
I	Case 1	$q_{1n}^* = \frac{M_1 - a_1\Delta_1}{2}$	$q_{2n}^* = \theta Q^* - \tau\gamma q_{1n}^*$
		$Q^* = \frac{\beta(M_2 - a_2 c_n) - a_2 c_o/\theta}{2\beta\theta} - \frac{(1-\tau)\gamma q_{1n}^*}{\theta}$	$q_{2r}^* = \gamma q_{1n}^*$
II	Case 2	$q_{1n}^* = \frac{M_1/a_1 - c_n - c_o + \beta\gamma(c_n - c_r) + \beta[(1-\tau)\gamma + \theta](M_2 - a_2 c_n)/a_2}{2/a_1 + 2\beta[(1-\tau)\gamma + \theta]^2/a_2}$	$q_{2n}^* = (\theta - \tau\gamma)q_{1n}^*$
		$Q^* = q_{1n}^*$	$q_{2r}^* = \gamma q_{1n}^*$
III	Cases 4-6	$q_{1n}^* = \frac{M_1 - a_1\Delta_0}{2}$	$q_{2n}^* = \Psi_1 - \gamma q_{1n}^*$
		$Q^* = q_{1n}^*$	$q_{2r}^* = \gamma q_{1n}^*$
IV	Case 7	$q_{1n}^* = \frac{M_1/a_1 - c_n - c_o + \beta\gamma(M_2 - a_2 c_r)/a_2}{2/a_1 + 2\beta\gamma^2/a_2}$	$q_{2n}^* = 0$
		$Q^* = q_{1n}^*$	$q_{2r}^* = \gamma q_{1n}^*$
V	Case 8	$q_{1n}^* = \frac{M_1 - a_1(c_n + c_o)}{2}$	$q_{2n}^* = 0$
		$Q^* = q_{1n}^*$	$q_{2r}^* = \Psi_2$

Table 3.4: Optimal solutions

parameter for the profitability of remanufacturing and for the optimal capacity and production quantities. As expected, a decrease in c_r always leads to more remanufacturing in period 2. Unless there already is a surplus of returns (case 8), this requires more returns and hence an increased production q_{1n}^* in the first period. For some cases, the reduction in manufacturing in period 2 is equal to the increase in remanufacturing, leaving the total production q_2^* in period 2 unchanged; for other cases total production increases. A particularly insightful result is that the capacity Q^* (in the first period) can either increase or decrease as a result of a reduction in c_r. To achieve the increased production in period 1 (so that more returns become available in period 2), an increased capacity is needed for cases where there is no capacity surplus in the first period. However, if a capacity surplus in the first period does exist and the increased remanufacturing leads to a lower capacity requirement in period 2 (since remanufacturing is less capacity intensive than manufacturing), then capacity can be reduced.

The effects of a change in the cost of manufacturing c_n are straightforward. An increased manufacturing cost leads to decreased production q_{1n}^* and capacity Q^* in the first period. Unless there is a surplus of returns as well as capacity (case 8), remanufacturing q_{2r}^* and total production q_2^* in period 2 also decrease. For all cases, manufacturing q_{2n}^* in period 2 is also non-increasing in c_n.

As expected, an increase in the cost of capacity c_o always leads to a reduced capacity Q^*. In most cases, the production q_{1n}^* in period 1 also decreases with c_o as less capacity is available. However, if there is a capacity surplus (case 1), then q_{1n}^* increases to compensate for the capacity shortage in period 2 (as this implies more returns and hence lower capacity needs in period 2). The compensation is only partial, as the total production in period 2 q_2^* decreases for case 1. For all other cases, q_2^* is also non-increasing in c_o.

A change in the relative capacity requirement for remanufacturing τ has little or no effect in most cases. With the exception of case 2, the total production q_2^* in period 2 is not affected by a change in τ. For most cases (3-8), this is explained by the fact that there is no shortage of capacity in period 2 and hence no benefit from a (further) reduced capacity needed in that period resulting from remanufacturing. For case 1, the reduced capacity need from a reduction in τ is beneficial, but is used to reduce the capacity Q^* in period 1 while keeping the total production in period 2 unchanged. For case 2, the reduced capacity need is also beneficial and indeed used to increase total production in period 2. For similar reasons, all policy parameters are unchanged for all cases except the first two.

The effects of an increase in the relative length θ of the second period are very similar to the just discussed effects of an increase in τ, as an increase in either parameter implies a higher capacity requirement (per produced unit) per time unit. To avoid a repetition of arguments, we instead refer to the above discussion on τ.

An increased return rate γ leads to higher availability of returns and increased remanufacturing in period 2, unless there is a surplus of returns (case 8). In some cases this also leads to increased total production in period 2, but in other cases manufacturing is reduced by the same amount by which remanufacturing increases. This is similar to the effect of a change in c_r discussed before, and the explanation why capacity Q^* can either increase or decrease with γ is along the same lines.

As expected, an increased market M_1 in period 1 always leads to increased production q_{1n}^* in that period. For all cases except case 1, this corresponding increase in capacity requirement also leads to increased capacity Q^*. For case 1 with a capacity surplus in period 1, Q^* decreases as more returns decrease the capacity requirement for period 2. The total production in period

2 either increases or is constant in M_1. So, as for a change in before discussed parameters, we see that an increase in the returns is sometimes used only to decrease cost as in cases 1 and cases 4-6, and in other cases also to increase production.

An increase in the market size M_2 for period 2 leads to increased total production for that period. This increase in production can be achieved by lowering the capacity requirement (through obtaining more returns) and/or by having more capacity available. This explains why both q_{1n}^* and Q^* are non-decreasing in M_2 for all cases.

The conducted sensitivity analysis also provides insights about the effect of remanufacturing on the capacity investment, production quantity and retail prices. For cases 1 and 4-6 the total production amount in period 2 does not change with remanufacturing parameters. That means retail prices in those cases are the same if there is no remanufacturing option in the second period. Thus, remanufacturing is used to reduce cost for cases 1 and 4-6, but not to increase production. However, for case 2, total production in the second period increases with the return rate. Consequently, retail prices with remanufacturing will be lower than that without remanufacturing for case 2.

3.5 Optimal policy if remanufacturing is either more costly or more capacity intensive

In this section, we relax the assumption that remanufacturing is both less expensive and less capacity intensive. Obviously, remanufacturing is not beneficial when it is more costly and more capacity intensive than only manufacturing. Thus, in this section we only consider the two scenarios that remanufacturing is either more costly but less capacity intensive, or less costly but more capacity intensive than manufacturing. For these two scenarios, we find the optimal policy for period 2 in Appendix 3.B and numerically study in Section 3.6 the effects on period 1 decisions and on the profitability of remanufacturing.

Similar to the analysis of the main case in Section 3.3, we first consider the second period problem and we obtain multiple regions with different optimality conditions with respect to first period decision parameters, q_{1n} and Q for both the cases where remanufacturing is either more costly or more capacity intensive. The analysis can be found in Appendix 3.B and the results

Region	Case		c_r	c_n	c_o	τ	θ	γ	M_1	M_2
I	Case 1	Q^*	$\nearrow$	$\searrow^{(1)}$	$\searrow$	$\nearrow$	$\frown$	$\searrow$	$\searrow$	$\nearrow$
		q_{1n}^*	$\searrow$	$\searrow$	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$	$\nearrow$	$\perp\!\!\!\perp$
		q_2^*	$\perp\!\!\!\perp$	$\searrow$	$\searrow$	$\perp\!\!\!\perp$	$\nearrow$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\nearrow$
		q_{2n}^*	$\nearrow$	$\searrow^{(2)}$	$\searrow$	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$
		q_{2r}^*	$\searrow$	$\searrow$	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$	$\nearrow$	$\perp\!\!\!\perp$
II	Case 2	Q^*	$\searrow$	$\searrow$	$\searrow$	$\frown$	$\frown$	$\frown$	$\nearrow$	$\nearrow$
		q_{1n}^*	$\searrow$	$\searrow$	$\searrow$	$\frown$	$\frown$	$\frown$	$\nearrow$	$\nearrow$
		q_2^*	$\searrow$	$\searrow$	$\searrow$			$\nearrow$	$\nearrow$	$\nearrow$
		q_{2n}^*	$\searrow$	$\searrow$	$\searrow$			$\frown$	$\nearrow$	$\nearrow$
		q_{2r}^*	$\searrow$	$\searrow$	$\searrow$	$\frown$		$\nearrow$	$\nearrow$	$\nearrow$
III	Case 4-6	Q^*	$\searrow$	$\searrow$	$\searrow$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\nearrow$	$\nearrow$	$\perp\!\!\!\perp$
		q_{1n}^*	$\searrow$	$\searrow$	$\searrow$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\nearrow$	$\nearrow$	$\perp\!\!\!\perp$
		q_2^*	$\perp\!\!\!\perp$	$\searrow^{(2)}$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\nearrow$
		q_{2n}^*	$\nearrow$	$\searrow$	$\nearrow$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\searrow$	$\searrow$	$\nearrow$
		q_{2r}^*	$\searrow$	$\searrow$	$\searrow$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\nearrow$	$\nearrow$	$\perp\!\!\!\perp$
IV	Case 7	Q^*	$\searrow$	$\searrow$	$\searrow$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\frown$	$\nearrow$	$\nearrow$
		q_{1n}^*	$\searrow$	$\searrow$	$\searrow$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\frown$	$\nearrow$	$\nearrow$
		q_2^*	$\searrow$	$\searrow$	$\searrow$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\nearrow$	$\nearrow$	$\nearrow$
		q_{2n}^*	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$
		q_{2r}^*	$\searrow$	$\searrow$	$\searrow$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\nearrow$	$\nearrow$	$\nearrow$
V	Case 8	Q^*	$\perp\!\!\!\perp$	$\searrow$	$\searrow$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\nearrow$	$\perp\!\!\!\perp$
		q_{1n}^*	$\perp\!\!\!\perp$	$\searrow$	$\searrow$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\nearrow$	$\perp\!\!\!\perp$
		q_2^*	$\searrow$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\nearrow$
		q_{2n}^*	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$
		q_{2r}^*	$\searrow$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\perp\!\!\!\perp$	$\nearrow$

Table 3.5: Impact of parameters on optimal solution ($\nearrow$: increasing, $\searrow$: decreasing, $\frown$: quasiconcave, $\perp\!\!\!\perp$: constant) (True for $c_n^{(1)}$ if $a_2 > a_1\gamma(1-\beta\gamma)(1-\tau)$ holds, true for $c_n^{(2)}$if $a_2 > a_1\gamma(1-\beta\gamma)$ holds.)

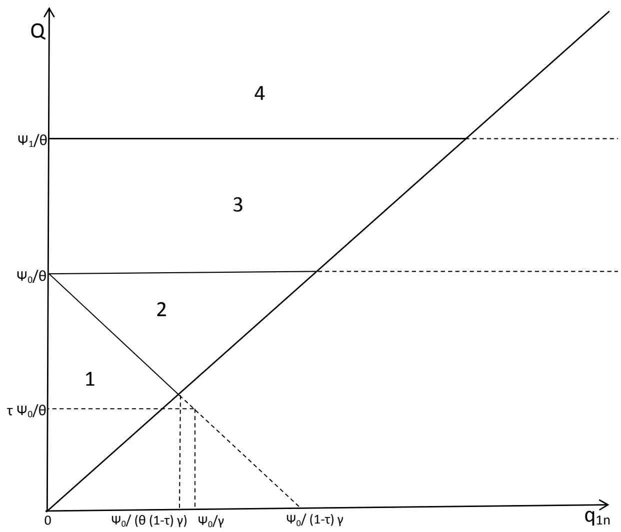

Figure 3.2: Optimality regions for the second period problem with respect to first period decisions when $c_n < c_r$

are presented graphically and discussed here.

We start with the scenario where remanufacturing is more costly but less capacity intensive. The optimality regions are depicted in Figure 3.2. In Region 1, it is optimal to both manufacture and remanufacture in the second period. Moreover, although remanufacturing is more expensive than manufacturing, it is optimal to remanufacture all available cores since there is insufficient capacity in the second period. In Region 2, it is also optimal to do both remanufacturing and manufacturing in the second period. However, there are more cores available and not all are remanufactured. In Regions 3 and 4, remanufacturing is not beneficial anymore.

Next, we consider the scenario that remanufacturing is less costly, but more capacity intensive. The optimality regions are depicted in Figure 3.3 and Figure 3.4 for the cases in which the conditions $\Psi_1/\Psi_0 > \tau > 1$ and $\tau > \Psi_1/\Psi_0 > 1$ hold respectively.

In Region 1, capacity is restricted and so it is not beneficial to remanufacture in the second period. In Region 2, the available capacity is sufficient to profitably remanufacture part but not all of the returns. In Region 3, capacity is high enough to make remanufacturing of all returns the preferred option; and further capacity increase leads to the situation in Region 4

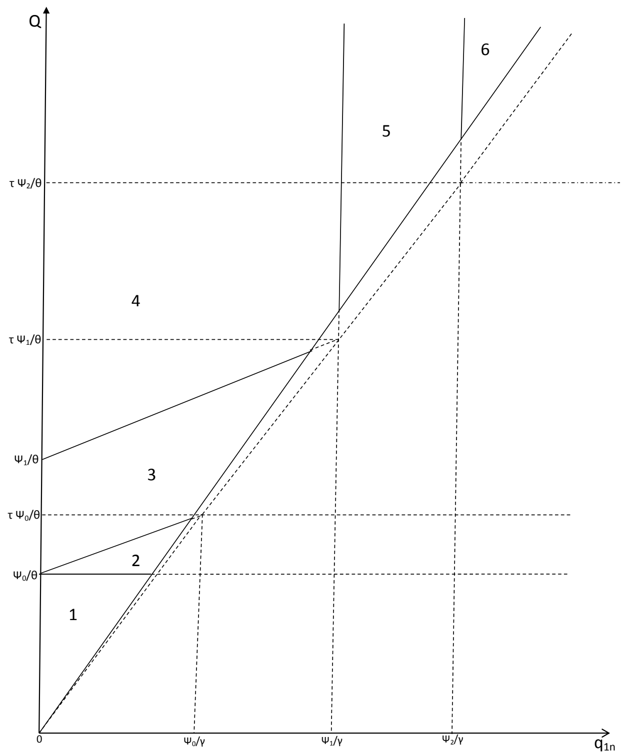

Figure 3.3: Optimality regions for the second period problem with respect to first period decisions when $\Psi_1/\Psi_0 > \tau > 1$ and $c_n > c_r$

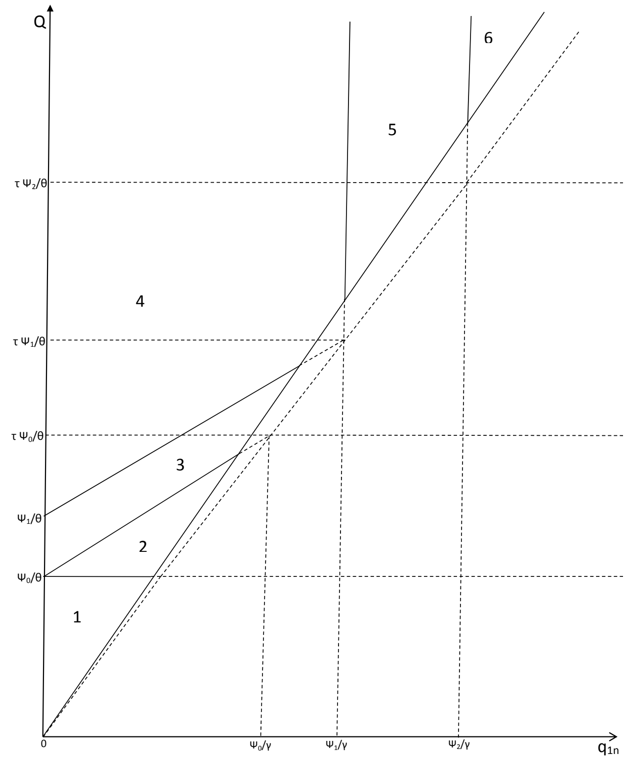

Figure 3.4: Optimality regions for the second period problem with respect to first period decisions when $\tau > \Psi_1/\Psi_0 > 1$ and $c_n > c_r$

where remanufacturing is restricted by the number of the available cores. In Region 5, capacity and cores availability are sufficient to fully replace manufacturing by remanufacturing; and in Region 6, it is not these availabilities but market conditions that are most restrictive for the production.

To summarize, for both scenarios there are parameter combinations (regions) where remanufacturing is applied and sometimes even "fully" replaces manufacturing, but this is not always the case.

3.6 Numerical analysis: Sensitivity and profitability for different scenarios

In the previous section, we consider different scenarios regarding to remanufacturing conditions and define the optimal production policy for a given capacity and production amount in the first period. The objective of this section is to obtain insights into the profitability of remanufacturing and its effects on capacity and production decisions across the different scenarios. Obviously, remanufacturing is most beneficial if remanufacturing is both less costly and less capacity intensive than manufacturing, but how much is lost if this is not the case? To answer this question, we perform an extensive numerical investigation where we separately consider the three relevant scenarios analyzed in previous sections and vary all relevant parameters, except for the discount factor that we fix at $\beta = 0.8$. For each scenario we consider a full factorial design with 59,049 different configurations for every possible combination of the parameter values presented in Table 3.6.

The numerical results are summarized in Tables 3.7, 3.8 and 3.9. Note that the regions are different for each scenario, as discussed in previous sections, and so results per region should not be compared across scenarios.

The first important result is that the relative profit gain is much smaller for the scenario where remanufacturing is more costly but less capacity intensive. Indeed, for this scenario, none of the regions show an average gain of more than 1%, and the average gain across all cases (in all regions, weighted for the number of cases in a region) is only 0.04%. On the other hand, the scenario where remanufacturing is less costly but more capacity intensive has a substantially

Parameter	Parameter values		
All scenarios			
M_1	10	55	100
M_2	10	55	100
a_1	0.1	0.6	1.1
a_2	0.1	0.6	1.1
c_o	0.1	2.5	4.9
γ	0.1	0.5	0.9
Remanufacturing less costly, less capacity intensive			
c_n	$c_r + 0.1$	$c_r + 2.1$	$c_r + 4.1$
c_r	0.1	2.5	4.9
τ	0.01	0.41	0.81
θ	1	2	3
Remanufacturing more costly, less capacity intensive			
c_n	0.1	2.5	4.9
c_r	$c_n + 0.1$	$c_n + 2.1$	$c_n + 4.1$
τ	0.01	0.41	0.81
θ	1	2	3
Remanufacturing less costly, more capacity intensive			
c_n	$c_r + 0.1$	$c_r + 2.1$	$c_r + 4.1$
c_r	0.1	2.5	4.9
τ	1.01	2.01	3.01
θ	2.8	3.8	4.8

Table 3.6: Parameters for numerical study

	Region1	Region2	Region3	Region4	Region5
Percentage of cases	25.07	2.62	49.86	3.35	19.10
Average percentage profit gain	1.22	8.15	1.02	7.04	0.45
Average $c_n - c_r$	2.12	2.08	2.05	2.82	2.07
Average τ	0.42	0.45	0.41	0.41	0.41
Average increase in capacity	-1.43	-1.49	-0.27	0.12	0

Table 3.7: Results if remanufacturing is less costly and less capacity intensive.

	Region1	Region2	Region3	Region4
Percentage of cases	8.76	2.87	24.79	63.55
Average percentage profit gain	0.45	0.14	0	0
Average $c_r - c_n$	0.63	0.64	2. 75	2. 11
Average τ	0.33	0.21	0.45	0.41
Average increase in capacity	-2.07	-2.24	0	0

Table 3.8: Results if remanufacturing is more costly and less capacity intensive.

	Region1	Region2	Region3	Region4	Region5	Region6
Percentage of cases	6.88	2.17	17.17	50.27	2.85	21.72
Average percentage profit gain	0	0.25	0.70	1.04	7.52	0.45
Average $c_n - c_r$	0.68	1.24	2.74	2.04	3.18	2.07
Average τ	2.63	2.86	1.82	1.98	2.01	2.01
Avg Increase in capacity	0	0.29	0.45	0.20	0.12	0

Table 3.9: Results if remanufacturing is less costly and more capacity intensive.

larger gain of 0.96% across all cases. Moreover, there is one region (numbered 5) with an average gain of more than 7%. Recall from Section 3.5 that this is the region where capacity in the second period is not a restricting factor, and so the increased capacity usage is not a disadvantage. Moreover, there is no manufacturing in period 2 and so the relative production cost savings are maximal by completely switching to remanufacturing. Also, and different from region 6, the market in period 2 is of sufficient size to remanufacture all the returns. Thus, we can state that remanufacturing can provide additional benefits even if it is either more costly or more capacity intensive than manufacturing but the additional benefit of remanufacturing is more pronounced when it is more capacity intensive but less costly, especially when there is sufficient capacity in the latter stage of the life-cycle.

For the scenario where remanufacturing offers benefits in terms lower cost and lower capacity usage, this same situation (capacity is not restrictive, all returns are remanufactured and no additional items are manufactured in period 2), now denoted as Region 4, also shows a considerable percentage gain of over 7% on average. A gain of similar size is observed for region 2. In this region, there is a capacity shortage in period 2 and so the decreased capacity usage from remanufacturing offers an additional advantage. The same holds for region 1, but that has a smaller market size and therefore fewer returns. The average gain across all cases is 1.35%.

3.7 Conclusions

Remanufacturing operations typically differ from manufacturing operations in both cost and capacity usage. In this chapter, we analyze their combined effects on production and capacity decisions and on the profitability of remanufacturing. The analysis is done for a two-period model with capacity and manufacturing decisions in the first period and manufacturing and remanufacturing decisions in the second period. We first consider the most preferred and realistic scenario where remanufacturing is less costly and less capacity intensive than manufacturing. Closed-form solutions are derived for all capacity and production decisions and used to conduct a sensitivity analysis. This revealed that in some cases, remanufacturing only replaces manufacturing to reduce cost leaving the total production in the second period unchanged. However, for other cases, remanufacturing does increase the total production quantity. A particularly interesting finding is that the availability of the less capital intensive remanufacturing option sometimes leads to an increased capital investment. In such cases, the additional investment is made to increase production in period 1 and thereby the number of returns in period 2. In other cases, with surplus capacity in the first period, introducing the remanufacturing option does lead to a reduction in the capacity investment.

We then relax the assumption that remanufacturing is less expensive and requires less capacity than manufacturing, and conduct a numerical study for the scenarios where remanufacturing is either less costly or less capacity intensive than manufacturing. It turned out that the scenario where remanufacturing is more costly is by far the least beneficial. Remanufacturing can retain considerable benefits if it is more capacity intensive, but only if the cost and market conditions are such that there is sufficient capacity in the latter stage of the life-cycle (period 2).

We conclude by pointing out three future research directions related to the limiting assumptions in our model. First of all, by relaxing the assumption of fixed return rate, we may consider a stochastic return rate due to the market uncertainty. Moreover, competition for collecting cores and or selling (re)manufactured products can be considered. Doing so would obviously require the inclusion of other players, such as collectors and/or OEMs. Furthermore, more than two-periods would allow a more accurate description of a product life-cycle.

Appendix 3.A. Optimality analysis for all eight cases

In this appendix, we determine the optimal values of q_{1n} and Q for all eight cases that are identified in Section 3.3 and described in Table 3.2. From that table, it is clear that the total production in period 2 is $(1-\tau)\gamma q_{1n} + \theta Q$ for cases 1 and 2, Ψ_1 for cases 3-6, γq_{1n} for case 7 and Ψ_2 for case 8. So $\Pi_2^*(q_{1n}, Q)$ can be rewritten as in Table 3.10 for different cases.

Using these results, (2) can be rewritten in terms of q_{1n} and Q for each case, after which the first order conditions can be derived to find the optimal solution. This is straightforward and the solutions are given in Table 3.4. We remark that the solutions for the different cases in Table 3.4 do not necessarily satisfy the case-dependent restrictions as listed in Table 3.3. Our next step is therefore to combine those restrictions into a single feasibility constraint for each separate case.

Case 1: From Table 3.2 we get

$$\begin{aligned} q_{1n} &< Q, \\ (1-\tau)\gamma q_{1n} + \theta Q &< \Psi_1, \\ \gamma q_{1n} &< \Psi_1. \end{aligned}$$

Note that if $(1-\tau)\gamma q_{1n} + \theta Q < \Psi_1$, we have $\gamma q_{1n} < \Psi_1$ since $\theta > \tau\gamma$. So we only need to consider the first two inequalities. Substituting the optimal solution from Table 3.4 we have

$$[\theta + (1-\tau)\gamma](M_1 - a_1\Delta_1) + a_2 c_o/(\beta\theta) < M_2 - a_2 c_n, \tag{3.3}$$

$$(M_2 - a_2 c_n)/2 - a_2 c_o/(\beta\theta) < (M_2 - a_2 c_n)/2. \tag{3.4}$$

Since the second inequality is clearly true, we conclude that this solution is feasible if (3.3) holds.

Case 2: From Table 3.2 we get

$$\begin{aligned} (1-\tau)\gamma q_{1n} + \theta Q &< \Psi_1, \\ \gamma q_{1n} &< \Psi_1. \end{aligned}$$

Similar to Case 1, we only need to consider the first inequality. Substituting the optimal solution from Table 3.4 we have

$$\begin{aligned} \frac{M_2 - a_2 c_n}{2} > \; & [(1-\tau)\gamma + \theta]\{M_1/a_1 - c_n - c_o + \beta\gamma(c_n - c_r) + \beta[(1-\tau)\gamma + \theta] \\ & \times (M_2/a_2 - c_n)\}\{2/a_1 + 2\beta[(1-\tau)\gamma + \theta]^2/a_2\}^{-1}, \end{aligned}$$

which simplifies to $M_2 - a_2 c_n > [(1-\tau)\gamma + \theta](M_1 - a_1\Delta_0)$.

Case 3: From Table 3.2 we get $q_{1n} < Q$. Note that $\gamma q_{1n} < \Psi_1$ since $(1-\tau)\gamma q_{1n} + \theta Q = \Psi_1$. Substituting the optimal solution from Table 3.4 we have

$$M_2 - a_2 c_n > [(1-\tau)\gamma + \theta](M_1 - a_1\Delta_1).$$

Case 4: We get $M_2 - a_2 c_n \geq 0$.

Case 5: From Table 3.2 we get $\gamma q_{1n} < \Psi_1 < (1-\tau)\gamma q_{1n} + \theta Q$. Substituting the optimal solution from Table 3.4 have

$$\gamma(M_1 - a_1\Delta_0) < M_2 - a_2 c_n < [(1-\tau)\gamma + \theta](M_1 - a_1\Delta_0).$$

Case 6: It is obvious from Table 3.2 that $\gamma q_{1n} = \Psi_1$. Substituting the optimal solution from Table 3.4 gives $\gamma(M_1 - a_1\Delta_0) = M_2 - a_2c_n$.

Case 7: From Table 3.2 we get $\gamma q^*_{1n} > \Psi_1$. Substituting the optimal solution from Table 4 gives

$$\gamma[M_1/a_1 - c_n - c_o + \beta\gamma(\frac{M_2 - a_2c_r}{a_2})](2/a_1 + 2\beta\gamma^2/a_2)^{-1} > \frac{M_2 - a_2c_n}{2},$$

which simplifies to $\gamma(M_1 - a_1\Delta_0) > M_2 - a_2c_n$.

Case 8: From Table 3.2 we get $\gamma q^*_{1n} > \Psi_2$. Substituting the optimal solution from Table 4 we have

$$\gamma\frac{M_1 - a_1(c_n + c_o)}{2} > \frac{M_2 - a_2c_r}{2},$$

which yields $M_2 - a_2c_r < \gamma[M_1 - a_1(c_n + c_o)]$.

Table 3.11 lists the feasibility constraints for all eight cases. Obviously, a certain case can only be optimal if it is feasible, i.e. the feasibility constraint in Table 3.11 is a necessary condition for optimality. However, it is not a sufficiency condition and, for some cases, we can derive further optimality constraints. This is done in the Appendix 3.A.1.

Appendix 3.A.1 Optimality conditions

From the case descriptions in Table 3.2 we see that case 2 is on the limit of case 1 in the following sense: case 1 considers all solutions with a positive (but possibly very small) surplus in the first period, whereas case 2 considers solution with a capacity match, i.e., zero surplus, in period 1. Note that we use the term limit rather than boundary, as we already use the latter term (in the main text) to describe (graphically - for M_1 versus M_2) neighboring regions. As case 2 is on the limit of case 1 and because the profit expression is continuous in all production quantities, the following must hold: if the optimal solution of case 1 is feasible, then it must have a higher profit than any solution of case 2. From the feasibility condition for case 1 in Table 3.11, we therefore get the following additional optimality condition for case 2: $M_2 - a_2c_n \leq [\theta + (1-\tau)\gamma](M_1 - a_1\Delta_1) + \frac{a_2c_o}{\beta\theta}$.

Similarly, case 4 is on the limit of both cases 3 and 5, which leads to the following additional optimality condition: $[(1-\tau)\gamma + \theta](M_1 - a_1\Delta_0) \leq M_2 - a_2c_n \leq [(1-\tau)\gamma + \theta](M_1 - a_1\Delta_1)$ (see Table 3.11).

Case	$\Pi_2^*(q_{1n}, Q)$
1	$\left(\frac{M_2-((1-\tau)\gamma q_{1n}+\theta Q)}{a_2} - c_n\right)((1-\tau)\gamma q_{1n} + \theta Q) + (c_n - c_r)\gamma q_{1n}$
2	$\left(\frac{M_2-((1-\tau)\gamma q_{1n}+\theta Q)}{a_2} - c_n\right)((1-\tau)\gamma q_{1n} + \theta Q) + (c_n - c_r)\gamma q_{1n}$
3	$\frac{l_n^2}{a_2} + (c_n - c_r)\gamma q_{1n}$
4	$\frac{l_n^2}{a_2} + (c_n - c_r)\gamma q_{1n}$
5	$\frac{l_n^2}{a_2} l_n + (c_n - c_r)\gamma q_{1n}$
6	$\frac{l_n^2}{a_2} + (c_n - c_r)\gamma q_{1n}$
7	$\left(\frac{M_2-\gamma q_{1n}}{a_2} - c_n\right)\gamma q_{1n} + (c_n - c_r)\gamma q_{1n}$
8	$\left(\frac{M_2-l_r}{a_2} - c_n\right) l_r + (c_n - c_r) l_r$

Table 3.10: $\Pi_2^*(q_{1n}, Q)$ for all cases.

Case	Feasibility constraint
1	$M_2 - a_2c_n > [\theta + (1-\tau)\gamma](M_1 - a_1\Delta_1) + \frac{a_2c_o}{\beta\theta}$
2	$[(1-\tau)\gamma + \theta](M_1 - a_1\Delta_0) < M_2 - a_2c_n$
3	$M_2 - a_2c_n > [(1-\tau)\gamma + \theta](M_1 - a_1\Delta_1)$
4	$M_2 - a_2c_n \geq 0$
5	$\gamma(M_1 - a_1\Delta_0) < M_2 - a_2c_n < [(1-\tau)\gamma + \theta](M_1 - a_1\Delta_0)$
6	$M_2 - a_2c_n = \gamma(M_1 - a_1\Delta_0)$
7	$M_2 - a_2c_n < \gamma(M_1 - a_1\Delta_0)$
8	$M_2 - a_2c_r > \gamma[M_1 - a_1(c_n + c_o)]$

Table 3.11: Feasibility of local optimal solutions for all cases.

Case	Feasibility constraint
1	$M_2 - a_2c_n > [\theta + (1-\tau)\gamma](M_1 - a_1\Delta_1) + \frac{a_2c_o}{\beta\theta}$
2	$[(1-\tau)\gamma + \theta](M_1 - a_1\Delta_0) < M_2 - a_2c_n \leq [\theta + (1-\tau)\gamma](M_1 - a_1\Delta_1) + \frac{a_2c_o}{\beta\theta}$
3	$M_2 - a_2c_n > [(1-\tau)\gamma + \theta](M_1 - a_1\Delta_1)$
4	$[(1-\tau)\gamma + \theta](M_1 - a_1\Delta_0) \leq M_2 - a_2c_n \leq [(1-\tau)\gamma + \theta](M_1 - a_1\Delta_1)$
5	$\gamma(M_1 - a_1\Delta_0) < M_2 - a_2c_n < [(1-\tau)\gamma + \theta](M_1 - a_1\Delta_0)$
6	$M_2 - a_2c_n = \gamma(M_1 - a_1\Delta_0)$
7	$M_2 - a_2c_n < \gamma(M_1 - a_1\Delta_0)$, $M_2 - a_2c_r \geq \gamma[M_1 - a_1(c_n + c_o)$
8	$M_2 - a_2c_r < \gamma[M_1 - a_1(c_n + c_o)$

Table 3.12: Restricted feasibility of optimal values for all cases.

By combining these additional conditions for cases 2 and 4 with the feasibility condition in Table 3.11, a stricter optimality condition is obtained. These are given in Table 3.12.

Appendix 3.A.2 Global optimal solution

Based on the optimality conditions in Table 3.12 we construct Figure 3.5. Figure 3.5 depicts local optimal solutions for regions defined with boundaries $L1, L2, L3, L4$, are given by

$$\begin{aligned}
M_2 - a_2c_n &= [(1-\tau)\gamma + \theta](M_1 - a_1\Delta_1) + \frac{a_2c_o}{\beta\theta}, \\
M_2 - a_2c_n &= [(1-\tau)\gamma + \theta](M_1 - a_1\Delta_1), \\
M_2 - a_2c_n &= [(1-\tau)\gamma + \theta](M_1 - a_1\Delta_0), \\
M_2 - a_2c_n &= \gamma(M_1 - a_1\Delta_0), \\
M_2 - a_2c_r &= \gamma(M_1 - a_1(c_n + c_o)).
\end{aligned}$$

What remains is to show for Regions I and II, which of the two candidates is indeed globally optimal. This is done next for each region separately. From (3.2) we determine the profit functions for all cases using Table 3.4 and list them in Table 3.13. We use superscript i to indicate the profit for case i.

Region I

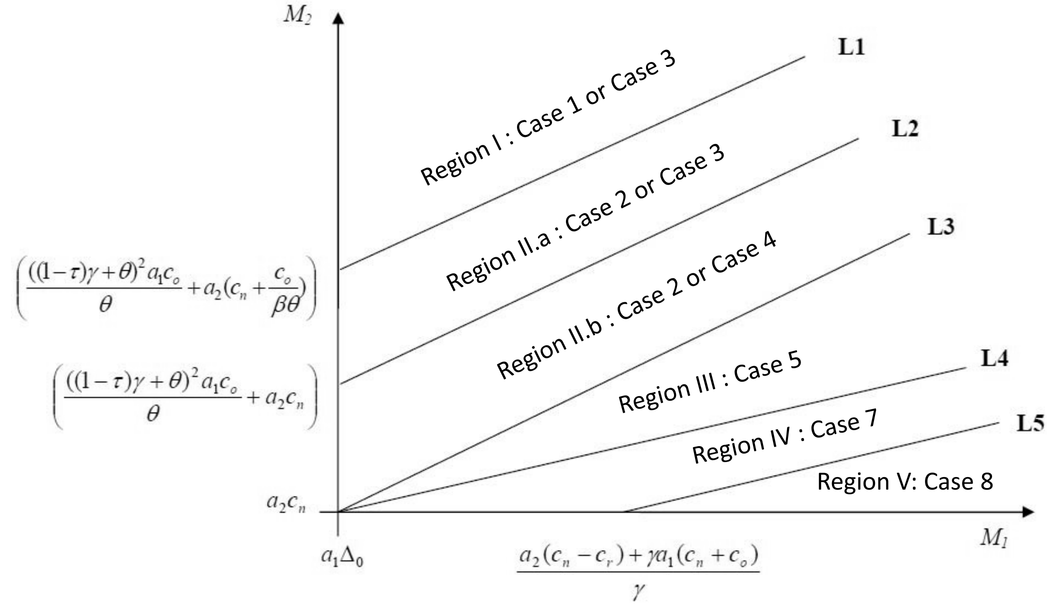

Figure 3.5: Comparing Cases (on line L3 case 4 is optimal and on line L4 case 6 is optimal)

Case	Profit
1	$\frac{(M_1-a_1\Delta_1)^2}{4a_1} + \beta\frac{(M_2-a_2c_n)^2}{4a_2} - c_o\frac{M_2-a_2c_n}{2\theta} + \frac{(a_2c_o^2)}{4\beta\gamma^2}$
2	$\frac{\{M_1/a_1-\Delta_0+\beta[(1-\tau)\gamma+\theta](M_2-a_2c_n)/a_2\}^2}{4/a_1+4\beta[(1-\tau)\gamma+\theta]^2/a_2}$
3	$\frac{(M_1-a_1\Delta_1)^2}{4a_1} + \beta\frac{(M_2-a_2c_n)^2}{4a_2} - c_o\frac{M_2-a_2c_n}{2\theta}$
4-6	$\left(\frac{M_1-a_1\Delta_0}{a_1} - \frac{M_2-a_2c_n}{2[(1-\tau)\gamma+\theta]a_1}\right)\left(\frac{M_2-a_2c_n}{2[(1-\tau)\gamma+\theta]}\right) + \beta\frac{(M_2-a_2c_n)^2}{4a_2}$
7	$\frac{(M_1/a_1-c_n-c_o+\beta\gamma(M_2-a_2c_r)/a_2)^2}{4(1/a_1+\beta\gamma^2/a_2)}$
8	$\frac{(M_1-a_1(c_o+c_n))^2}{4a_1} + \beta\frac{(M_2-a_2c_r)^2}{4a_2}$

Table 3.13: Profit functions of the 8 Cases

From Table 3.13 we can easily get

$$\Pi^1 - \Pi^3 = \frac{(a_2 c_o^2)}{4\beta\gamma^2} > 0.$$

Region II.a

The optimality condition for this region is

$$[(1-\tau)\gamma + \theta](M_1 - a_1\Delta_1) < M_2 - a_2 c_n \leq [(1-\tau)\gamma + \theta](M_1 - a_1\Delta_1) + \frac{a_2 c_o}{\beta\theta}. \tag{3.5}$$

Let us define $M_2 - a_2 c_n$ by $x, (1-\tau)\gamma + \theta$ by A, and define $f(x) = \Pi^2 - \Pi^3$. Then, from Table 3.13, we have

$$f(x) = -\frac{\beta x^2}{a_2 + \beta A^2 a_1} + [\frac{2\beta A(M_1 - a_1\Delta_0)}{a_2 + \beta A^2 a_1} + \frac{2c_o}{\theta}]x + \frac{a_2(M_1 - a_1\Delta_0)^2}{a_1(a_2 + \beta A^2 a_1)} - \frac{(M_1 - a_1\Delta_1)^2}{a_1},$$

which clearly is strictly concave in x.

Differentiating $f(x)$ with respect to x, we have

$$\begin{aligned} f'(x) &= -\frac{2\beta x}{a_2 + \beta A^2 a_1} + [\frac{2\beta A(M_1 - a_1\Delta_0)}{a_2 + \beta A^2 a_1} + \frac{2c_o}{\theta}] \\ &\sim -\frac{x}{a_2 + \beta A^2 a_1} + [\frac{A(M_1 - a_1\Delta_0)}{a_2 + \beta A^2 a_1} + \frac{c_o}{\beta\theta}], \\ &= -\frac{x}{a_2 + \beta A^2 a_1} + \frac{A(M_1 - a_1\Delta_1)}{a_2 + \beta A^2 a_1} + \frac{c_o a_2/\beta\theta}{a_2 + \beta A^2 a_1}. \end{aligned}$$

By (3.5), we have that $f'(x)$ is non-negative.

Since $A = (1-\tau)\gamma + \theta$, we have $\Delta_0 = \Delta_1 + c_o A/\theta$. Further denoting $y = M_1 - a_1\Delta_1$, we get

$$\begin{aligned} f(A(M_1 - a_1\Delta_1)) &= -\frac{(a_2 + 2a_1\beta A^2)y^2}{a_1(a_2 + \beta A^2 a_1)} + \frac{a_2(y - a_1 A c_o/\theta)^2}{a_1(a_2 + \beta A^2 a_1)} + [\frac{2\beta A(y - a_1 A c_o/\theta)}{a_2 + \beta A^2 a_1} + \frac{2c_o}{\theta}]Ay \\ &= \frac{a_2 a_1 A^2 c_o^2}{\theta^2(a_2 + \beta A^2 a_1)} > 0. \end{aligned}$$

Thus, we have $\Pi^2 > \Pi^3$ in this region.

Region II

Denote $A = (1-\tau)\gamma + \theta$, $C = M_2 - a_2 c_n$ and $D = M_1 - a_1\Delta_0$. From Table 3.13 we have

$$\Pi^4 = \frac{DC}{2Aa_1} - \frac{C^2}{4A^2 a_1} + \beta\frac{C^2}{4a_2},$$

and

$$\Pi^2 = \frac{(\frac{D}{a_1} - \frac{\beta AC}{a_2})^2}{4/a_1 + 4\beta A^2/a_2}.$$

Then, we have

$$\Pi^2 - \Pi^4 = \frac{\frac{D^2}{a_1^2} - \frac{2DC}{Aa_1^2} + \frac{C^2}{A^2a_1^2}}{4/a_1 + 4\beta A^2/a_2} = \frac{(1/a_1^2)(D - C/A)^2}{4/a_1 + 4\beta A^2/a_2} > 0.$$

Thus, case 2 is always better than case 4 in Region III.

Appendix 3.B. Optimality analysis when remanufacturing is either more costly or more capacity intensive

In this appendix we relax the assumption that remanufacturing is both less expensive and requires less capacity than manufacturing and consider two different scenarios. At first we look at the scenario in which remanufacturing is less costly but more capacity intensive than manufacturing, and later we investigate the conditions where remanufacturing requires less capacity than manufacturing but it is more expensive. For both scenarios the Lagrangian relaxation is given by

$$\min_{q_2, q_{2r}, \lambda_i} \{-(\frac{M_2 - q_2}{a_2} - c_n)q_2 - (c_n - c_r)q_{2r} + \lambda_1[q_2 - (1-\tau)q_{2r} - \theta Q] + \lambda_2(q_{2r} - \gamma q_{1n})\}, \quad (3.6)$$

where λ_1 and λ_2 are non-negative and Lagrangian multipliers.

The first-order and complementary slackness conditions are given by

$$-\frac{M_2 - a_2c_n - 2q_2}{a_2} + \lambda_1 = 0, \quad (3.7)$$

$$-(c_n - c_r) - \lambda_1(1-\tau) + \lambda_2 = 0, \quad (3.8)$$

$$\lambda_1[q_2 - (1-\tau)q_{2r} - \theta Q] = 0, \quad (3.9)$$

$$\lambda_2(q_{2r} - \gamma q_{1n}) = 0. \quad (3.10)$$

Appendix 3.B.1: Remanufacturing more costly

At first, we look at the scenario when remanufacturing is more costly than manufacturing i.e. $c_r > c_n$. First of all observe that $\lambda_1 = 0$ cannot be zero since (3.8) would otherwise give $\lambda_2 < 0$ which cannot hold since both λ_1 and λ_2 have to be non-negative. Thus, we only consider two cases; $\lambda_2 = 0$ and $\lambda_2 > 0$.

Case 1: $\lambda_2 = 0$.

From (3.8) we have

$$\lambda_1 = \frac{c_r - c_n}{1-\tau} > 0. \tag{3.11}$$

By substituting (3.11) into (3.7), we obtain

$$q_2 = \frac{M_2 - a_2 c_n - a_2\lambda_1}{2} = \frac{M_2 - a_2 c_n}{2} - \frac{a_2(c_r - c_n)}{2(1-\tau)}. \tag{3.12}$$

And by substituting (3.12) into (3.9), we have

$$q_{2r} = \frac{q_2 - \theta Q}{1-\tau} = \frac{1}{2(1-\tau)}[M_2 - a_2 c_n - \frac{a_2(c_r - c_n)}{1-\tau} - 2\theta Q]. \tag{3.13}$$

Note that the number of remanufactured products has to be nonnegative and restricted by the amount of available cores. In other words $0 \leq q_{2r} \leq \gamma q_{1n}$ must hold, which gives two inequalities, (3.14) and (3.15). Additionally, non-negativity of q_{2n} provides one more inequality (3.16).

$$\theta Q - \Psi_0 \leq 0, \tag{3.14}$$

$$\Psi_0 - \theta Q - (1-\tau)\gamma q_{1n} \leq 0, \tag{3.15}$$

$$\tau\Psi_0 - \theta Q \leq 0. \tag{3.16}$$

If (3.14), (3.15) and (3.16) all hold then the optimal solution satisfies (3.12) and (3.13), which gives $q_2^* = \Psi_0$, $q_{2r}^* = (\Psi_0 - \theta Q)/(1-\tau)$, and $q_{2n}^* = (\theta Q - \tau\Psi_0)/(1-\tau)$.

If (3.14) does not hold then $q_{2r}^* = 0$ and conditions (3.15) and (3.16) automatically hold. If $\Psi_1 \leq \theta Q$ then $q_{2n}^* = \Psi_1$; otherwise, if $\Psi_0 \leq \theta Q \leq \Psi_1$ then $q_{2n}^* = \theta Q$.

If (3.14) and (3.15) hold but (3.16) does not hold, then $q_{2n}^* = 0$. Hence $q_2 = q_{2r}$ so that (3.9) gives $q_{2r}^* = \theta Q/\tau$. Since we assume that (3.15) holds, $\gamma q_{1n} > q_{2r}^* = \theta Q/\tau$ will always hold. Note that we also consider the capacity constraint for the first period, i.e., $q_{1n} \leq Q$, and for the entire analysis we assume $\gamma\tau \leq \theta$. These two inequalities contradict with $\gamma q_{1n} > \theta Q/\tau$. Thus, the case $q_{2n}^* = 0$ and $q_{2r}^* = \theta Q/\tau$ is never optimal.

If we consider $q_{2n} = 0$ in (3.6) when (3.14) and (3.15) hold but (3.16) does not hold, we will obtain $q_{2r} = \Psi_2$. The capacity constraint in the second period implies that $\tau\Psi_2 \leq \theta Q$ must hold. Since $\tau\Psi_0 < \tau\Psi_2$ holds, this contradicts with our assumption that $\theta Q < \tau\Psi_0$. Thus, $q_{2r} = \Psi_2$ is never optimal.

Case 2: $\lambda_2 > 0$**.**

By (3.10) we have $q_{2r} = \gamma q_{1n}$. If we substitute this into (3.9) we obtain $q_2 = (1-\tau)q_{2r} + \theta Q = (1-\tau)\gamma q_{1n} + \theta Q$. By (3.8) we have $\lambda_1 = (c_r - c_n + \lambda_2)/(1-\tau) > 0.$. By (3.7), we have $\lambda_1 = M_2 - a_2 c_n - 2[(1-\tau)\gamma q_{1n} + \theta Q]/a_2$ and $\lambda_2 = \lambda_1 - (c_r - c_n)/(1-\tau)$.

Note that we need to check for the non-negativity conditions for λ_1 and λ_2. From the non-negativity conditions, we obtain the following two inequalities,

$$\Psi_1 > (1-\tau)\gamma q_{1n} + \theta Q, \tag{3.17}$$

$$\Psi_0 > (1-\tau)\gamma q_{1n} + \theta Q. \tag{3.18}$$

Condition (3.17) always holds since $\Psi_1 > \Psi_0$. Thus, when (3.18) holds the optimal solution is given by $q_2^* = (1-\tau)\gamma q_{1n} + \theta Q$, $q_{2r}^* = \gamma q_{1n}$, and $q_{2n}^* = \theta Q - \tau\gamma q_{1n}$.

Appendix 3.B.2: Remanufacturing more capacity intensive

From (3.8) it is clear that the case $\lambda_1 = \lambda_2 = 0$ cannot hold since we assume that $c_n > c_r$ holds. Thus, we have to consider three other cases. The first occurs when $\lambda_1 > 0$ and $\lambda_2 = 0$, the second case where $\lambda_1 > 0$ and $\lambda_2 > 0$ and finally we analyze the case when $\lambda_1 = 0$ and $\lambda_2 > 0$.

Case 1: $\lambda_1 > 0$, $\lambda_2 = 0$**.**

By (3.8) we have

$$\lambda_1 = \frac{c_n - c_r}{\tau - 1} > 0. \tag{3.19}$$

By substituting (3.19) into (3.7), we obtain

$$q_2 = \frac{M_2 - a_2 c_n}{2} - \frac{a_2(c_n - c_r)}{2(\tau - 1)}. \tag{3.20}$$

And by substituting (3.20) into (3.9), we have

$$q_{2r}^* = \frac{\theta Q - \frac{M_2 - a_2 c_n}{2} - \frac{a_2(c_n - c_r)}{2(\tau-1)}}{\tau - 1} = \frac{\theta Q - \Psi_0}{(\tau - 1)}. \tag{3.21}$$

Note that the number of remanufactured products has to be nonnegative and is restricted by the amount of available cores. In other words $0 \leq q_{2r} \leq \gamma q_{1n}$ must hold which gives inequalities (3.22) and (3.23). Additionally, non-negativity of q_{2n} provides inequality (3.24).

$$\theta Q - \Psi_0 \geq 0, \tag{3.22}$$

$$\theta Q - \Psi_0 \leq (\tau - 1)\gamma q_{1n}, \tag{3.23}$$

$$\tau\Psi_0 - \theta Q \geq 0. \tag{3.24}$$

If (3.22), (3.23) and (3.24) all hold, then, the optimal solution satisfies (3.20) and (3.21) which can be rewritten as $q_2^* = \Psi_0$, $q_{2r}^* = (\theta Q - \Psi_0)/(\tau - 1)$, and $q_{2n}^* = (\tau\Psi_0 - \theta Q)/(\tau - 1)$.

If (3.22) does not hold, then (3.23) and (3.24) must hold so that $q_{2r}^* = 0$ and $q_{2n}^* = \theta Q$.

Case 2 :$\lambda_1 = 0,\ \lambda_2 > 0$**.**

By (3.10) we have $q_{2r}^* = \gamma q_{1n}$ and (3.8) gives $\lambda_2 = c_n - c_r > 0$. By (3.7) we have $q_2 = \Psi_1$ and $q_{2n} = \Psi_1 - \gamma q_{1n}$.

This is the optimal solution for the second period problem for given q_{1n} and Q value if the following inequalities hold.

$$\Psi_1 \geq \gamma q_{1n}, \tag{3.25}$$

$$(\tau - 1)\gamma q_{1n} + \Psi_1 \leq \theta Q. \tag{3.26}$$

If (3.25) does not hold, i.e when $\Psi_2 \geq \gamma q_{1n} \geq \Psi_1$, then $q_{2n}^* = 0$ and $q_{2r}^* = \gamma q_{1n}$ is the optimal solution assuring that $\tau\gamma q_{1n} \leq \theta Q$ also holds.

Also, if we consider (3.6) when $q_{2n}^* = 0$ under the condition that (3.25) does not hold, the optimal solution will be $q_{2r}^* = \Psi_2$, when $\gamma q_{1n} \geq \Psi_2$ and $\theta Q \geq \tau\Psi_2$ hold.

Case 3 :$\lambda_1 > 0,\ \lambda_2 > 0$**.**

By (3.9) and (3.10) we have $q_2^* = \theta Q - (\tau - 1)\gamma q_{1n}$, $q_{2r}^* = \gamma q_{1n}$, and $q_{2n}^* = \theta Q - \tau\gamma q_{1n}$.

To ensure that λ_1 and λ_2 are positive, $\Psi_0 \leq \theta Q - (\tau - 1)\gamma q_{1n} \leq \Psi_1$ must hold.

Appendix 3.C. Optimal solution in the region $(M_1, M_2 \geq 0)$

In the main text, we restrict our attention to the situation where $M_1 \geq a_1\Delta_0$ and $M_2 \geq a_2 c_n$. Now we relax these two assumptions and present the global optimal solution in the whole region $(M_1, M_2 \geq 0)$. Since the analysis depends on whether or not $\Delta_1 \geq 0$, we first assume $\Delta_1 \geq 0$ and later address the situation that $\Delta_1 < 0$. Figure 3.6 shows the global optimal solutions with 8 regions. The detailed reasoning is given below.

Region I

By Table 3.4, we note that q_{1n}^* in Region I is non-negative as long as $M_1 \geq a_1\Delta_1$. So, we can extend Region I till $M_1 = a_1\Delta_1$.

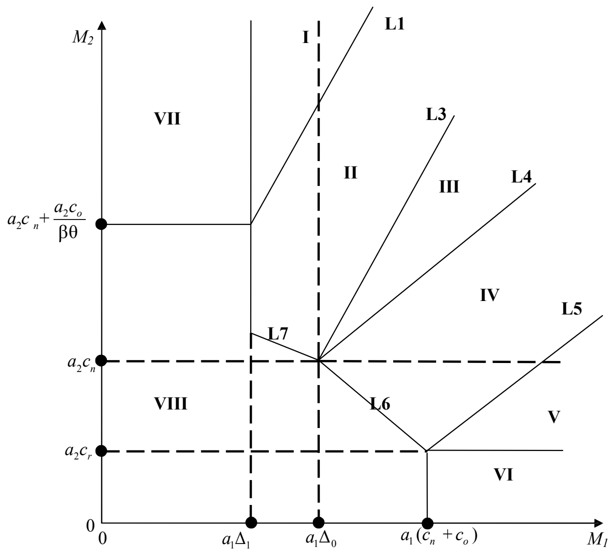

Figure 3.6: The global optimal solution for $\Delta_1 \geq 0$.

Region II

By Table 3.4, we note that the optimal q_{1n}^* in Region II still applies as long as

$$\frac{M_1 - a_1\Delta_0}{a_1} + \beta[(1-\tau)\gamma + \theta]\frac{M_2 - a_2c_n}{a_2} \geq 0.$$

So, we can extend Region II till the boundary by $L7$, where $L7$ is given by

$$\frac{M_1 - a_1\Delta_0}{a_1} + \beta[(1-\tau)\gamma + \theta]\frac{M_2 - a_2c_n}{a_2} = 0.$$

Further, by the optimality condition of case 2, we have $q_{1n}^* \leq \frac{M_1 - a_1\Delta_1}{2}$. Since $q_{1n}^* \geq 0$, we also have to require that $M_1 \geq a_1\Delta_1$, as for Region I.

Region III

Region III remains the same since it requires $M_1 \geq a_1\Delta_0$ and $M_2 \geq a_2c_n$.

Region IV

By Table 3.4, we note that q_{1n}^* in Region IV is still true as long as

$$\frac{M_1 - a_1(c_n + c_o)}{a_1} + \beta\gamma\frac{M_2 - a_2c_n}{a_2} \geq 0.$$

So, we can extend Region V till the boundary by $L6$, where $L6$ is given by

$$\frac{M_1 - a_1(c_n + c_o)}{a_1} + \beta\gamma\frac{M_2 - a_2c_n}{a_2} = 0.$$

Region V

By Table 3.4, we can extend Region V till $M_1 = a_1(c_n + c_o)$ and $M_2 = a_2 c_r$.

Region VI

We intend to show that in Region VI, the firm only produces in period 1. The reasoning is as follows. In period 2, it is clear that the maximal price is M_2/a_2. If $M_2/a_2 \leq c_r$, the firm does not have any incentive to perform production in period 2. Under this situation, the firm may only produce in period 1. Then, we can rewrite (3.2) as follows,

$$\Pi_1(q_{1n}, Q) = (p_1 - c_n)q_{1n} - c_o Q. \tag{3.27}$$

Since $\Pi_1(q_{1n}, Q)$ is jointly concave in (q_{1n}, Q), it is obvious that $q_{1n}^* = \frac{M_1 - a_1(c_n + c_o)}{2}$ and $Q^* = q_{1n}^*$. Since $q_{1n}^* \geq 0$ when $M_1 \geq a_1(c_n + c_o)$, the optimal solution is feasible. So, in Region VI, the firm only produces in period 1.

Region VII

We intend to show that in Region VII, the firm only produces in period 2. The corresponding profit function is given by

$$\Pi_1(q_{2n}, Q) = \beta(p_2 - c_n)q_{2n} - c_o Q. \tag{3.28}$$

Since $\Pi_1(q_{2n}, Q)$ is jointly concave in (q_{2n}, Q), it is obvious that $q_{2n}^* = \frac{M_2 - a_2(c_n + c_o) - a_2 c_o/(\beta\theta)}{2}$ and $Q^* = q_{2n}^*/\theta$. Since $q_{1n}^* \geq 0$, when $M_2 \geq a_2(c_n + c_o) + a_2 c_o/(\beta\theta)$, the optimal solution is feasible. So, in Region VII, the firm only produces in period 2.

Region VIII

Finally, in Region VIII, since neither M_1 nor M_2 is large enough, the firm produces in neither period.

Now we discuss how these results change if $\Delta_1 < 0$. Figure 3.7 shows a scenario where the intercept of $L1$ is less than that of $L7$, i.e.,

$$\frac{a_2 c_o}{\beta\theta} - a_1\Delta_1[(1-\tau)\gamma + \theta] < \frac{a_1\Delta_0}{\beta[(1-\tau)\gamma + \theta]}.$$

Compared with Figure 3.6, the main difference is that Region VII disappears since $\Delta_1 < 0$ (as the unit capacity cost saving $(c_o(1-\tau)\gamma/\theta)$ outweighs the unit production cost $(c_n - \beta\gamma(c_n - c_r))$ in period 1), which indicates that it is always profitable to produce in period 1. Also, the intercept of $L1$ is at least that of $L7$ and so it only changes the areas of Region I and II.

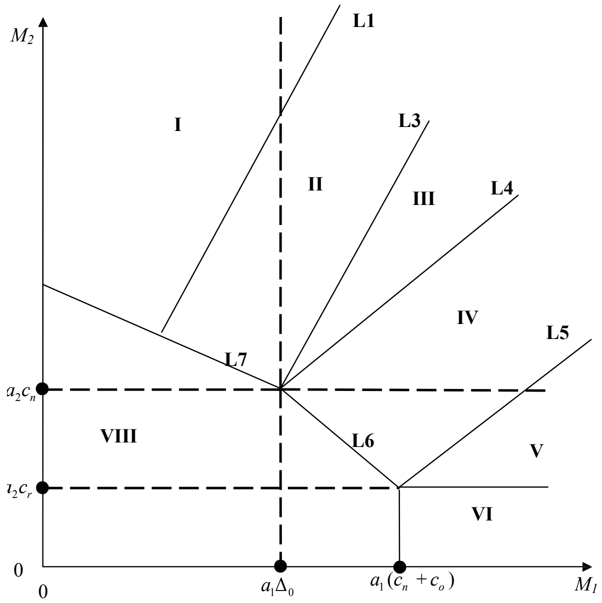

Figure 3.7: The global optimal solution for $\Delta_1 < 0$.

Appendix 3.D. Sensitivity analysis

In this section we analyze the effect of parameters on the optimal solution.

Appendix 3.D.1 Sensitivity analysis with respect to c_r

Case 1

$$
\begin{aligned}
\frac{\partial q_{1n}^*}{\partial c_r} &= -\frac{a_1\beta\gamma}{2} < 0,\\
\frac{\partial Q^*}{\partial c_r} &= -\frac{(1-\tau)\gamma}{\theta}\frac{\partial q_{1n}^*}{\partial c_r} = \frac{(1-\tau)a_1\beta\gamma^2}{2\theta} > 0,\\
\frac{\partial q_{2r}^*}{\partial c_r} &= \gamma\frac{\partial q_{1n}^*}{\partial c_r} = -\frac{a_1\beta\gamma^2}{2} < 0, \frac{\partial q_{2n}^*}{\partial c_r} = \theta\frac{\partial Q^*}{\partial c_r} - \tau\gamma\frac{\partial q_{1n}^*}{\partial c_r} = \frac{a_1\beta\gamma^2}{2} > 0,\\
\frac{\partial q_2^*}{\partial c_r} &= \frac{\partial q_{2n}^*}{\partial c_r} + \frac{\partial q_{2r}^*}{\partial c_r} = 0.
\end{aligned}
$$

Thus, as c_r increases, q_{1n}^* and q_{2r}^* decrease. Q^* and q_{2n}^* increase and q_2^* does not change.

Case 2

$$\begin{aligned}
\frac{\partial Q^*}{\partial c_r} &= \frac{\partial q_{1n}^*}{\partial c_r} = -\frac{\beta\gamma}{2/a_1 + 2\beta[(1-\tau)\gamma+\theta]^2/a_2} < 0,\\
\frac{\partial q_{2r}^*}{\partial c_r} &= \gamma\frac{\partial q_{1n}^*}{\partial c_r} < 0, \frac{\partial q_{2n}^*}{\partial c_r} = (\theta-\tau\gamma)\frac{\partial q_{1n}^*}{\partial c_r} < 0,\\
\frac{\partial q_2^*}{\partial c_r} &= [\theta+(1-\tau)\gamma]\frac{\partial q_{1n}^*}{\partial c_r} < 0.
\end{aligned}$$

Thus, as c_r increases, q_{1n}^*, Q^*, q_{2n}^*, q_{2r}^* and q_2^* decrease.

Case 4-6

$$\begin{aligned}
\frac{\partial Q^*}{\partial c_r} &= \frac{\partial q_{1n}^*}{\partial c_r} = -\frac{a_1\beta\gamma}{2} < 0, \frac{\partial q_{2r}^*}{\partial c_r} = \gamma\frac{\partial q_{1n}^*}{\partial c_r} < 0,\\
\frac{\partial q_{2n}^*}{\partial c_r} &= -\gamma\frac{\partial q_{1n}^*}{\partial c_r} > 0, \frac{\partial q_2^*}{\partial c_r} = 0.
\end{aligned}$$

Thus, as c_r increases, Q^*, q_{1n}^* and q_{2r}^* decrease, q_{2n}^* increases and q_2^* does not change.

Case 7

$$\begin{aligned}
\frac{\partial Q^*}{\partial c_r} &= \frac{\partial q_{1n}^*}{\partial c_r} = -\frac{\beta\gamma}{2/a_1 + 2\beta\gamma^2/a_2} < 0,\\
\frac{\partial q_2^*}{\partial c_r} &= \frac{\partial q_{2r}^*}{\partial c_r} = \gamma\frac{\partial q_{1n}^*}{\partial c_r} < 0, \frac{\partial q_{2n}^*}{\partial c_r} = 0.
\end{aligned}$$

Thus, as c_r increases, q_{1n}^*, Q^*, q_{2n}^* ,q_{2r}^* decrease.

Case 8

$$\frac{\partial q_2^*}{\partial c_r} = \frac{\partial q_{2r}^*}{\partial c_r} = -\frac{a_2}{2} < 0.$$

So Q, q_{1n}^*, q_{2n}^* are constant with respect to c_r,and q_{2r}^* decreases. with c_r.

Appendix 3.D.2 Sensitivity analysis with respect to c_n

Case 1

$$\begin{aligned}
\frac{\partial q_{1n}^*}{\partial c_n} &= -\frac{a_1(1-\beta\gamma)}{2} < 0,\\
\frac{\partial Q^*}{\partial c_n} &= -\frac{a_2}{2\theta} + \frac{a_1(1-\beta\gamma)\,(1-\tau)\,\gamma}{2\theta},\\
\frac{\partial q_{2r}^*}{\partial c_n} &= \gamma\frac{\partial q_{1n}^*}{\partial c_n} = -\frac{a_1\gamma(1-\beta\gamma)}{2} < 0,\\
\frac{\partial q_{2n}^*}{\partial c_n} &= \theta\frac{\partial Q^*}{\partial c_n} - \tau\gamma\frac{\partial q_{1n}^*}{\partial c_n} = -\frac{a_2}{2} + \frac{a_1(1-\beta\gamma)\gamma}{2},\\
\frac{\partial q_2^*}{\partial c_n} &= \frac{\partial q_{2n}^*}{\partial c_n} + \frac{\partial q_{2r}^*}{\partial c_n} = \frac{a_2}{2} < 0.
\end{aligned}$$

Thus, q_{1n}^*, q_{2r}^*, q_2^* decrease in c_r. If $a_2 > a_1(1-\beta\gamma)(1-\tau)\gamma$, then Q^* decreases in c_n; otherwise, Q^* increases in c_n. If $a_2 > a_1(1-\beta\gamma)\gamma$ holds, q_{2n}^* decreases in c_n; otherwise, q_{2n}^* increases in c_n.

Case 2

$$\begin{aligned}
\frac{\partial Q^*}{\partial c_n} &= \frac{\partial q_{1n}^*}{\partial c_n} = \frac{\beta(\tau\gamma-\theta)-1}{2/a_1+2\beta[(1-\tau)\gamma+\theta]^2/a_2} < 0,\\
\frac{\partial q_{2r}^*}{\partial c_n} &= \gamma\frac{\partial q_{1n}^*}{\partial c_n} < 0, \frac{\partial q_{2n}^*}{\partial c_n} = (\theta-\tau\gamma)\frac{\partial q_{1n}^*}{\partial c_n} < 0,\\
\frac{\partial q_2^*}{\partial c_n} &= [\theta+(1-\tau)\gamma]\frac{\partial q_{1n}^*}{\partial c_n} < 0.
\end{aligned}$$

We have $\beta(\tau\gamma-\theta) < 0$ since $\tau\gamma \le \theta$ holds. Thus, as c_n increases, q_{1n}^*, Q^*, q_{2n}^*, q_{2r}^* and q_2^* all decrease.

Case 4-6

$$\begin{aligned}
\frac{\partial Q^*}{\partial c_n} &= \frac{\partial q_{1n}^*}{\partial c_n} = -\frac{a_1(1-\beta\gamma)}{2} < 0, \frac{\partial q_{2r}^*}{\partial c_n} = \gamma\frac{\partial q_{1n}^*}{\partial c_n} < 0,\\
\frac{\partial q_{2n}^*}{\partial c_n} &= -\frac{a_2}{2} + \frac{a_1(1-\beta\gamma)\gamma}{2}, \frac{\partial q_2^*}{\partial c_n} = -\frac{a_2}{2} < 0.
\end{aligned}$$

Thus, as c_n increases, q_{1n}^*, q_{2r}^*, q_2^* and Q^* all decrease. If $a_2 > a_1(1-\beta\gamma)\gamma$, then q_{2n}^* decreases in c_n; otherwise, q_{2n}^* increases in c_n.

Case 7

$$\begin{aligned}
\frac{\partial Q^*}{\partial c_n} &= \frac{\partial q_{1n}^*}{\partial c_n} = -\frac{1}{2/a_1+2\beta\gamma^2/a_2} < 0,\\
\frac{\partial q_2^*}{\partial c_n} &= \frac{\partial q_{2r}^*}{\partial c_n} = \gamma\frac{\partial q_{1n}^*}{\partial c_n} < 0, \frac{\partial q_{2n}^*}{\partial c_n} = 0.
\end{aligned}$$

Thus, as c_n increases, q_{1n}^*, q_{2r}^*, q_2^* and Q^* decrease.

Case 8

$$\frac{\partial q_{1n}^*}{\partial c_n} = \frac{\partial Q^*}{\partial c_n} = -\frac{a_1}{2} < 0.$$

Thus, as c_n increases, q_{1n}^* and Q^* decrease and q_{2n}^*, q_{2r}^*, q_2^* is constant with respect to c_n.

Appendix 3.D.3 Sensitivity analysis with respect to c_o

Case 1

$$\begin{aligned}
\frac{\partial q_{1n}^*}{\partial c_o} &= \frac{a_1(1-\tau)\gamma}{\theta} > 0,\\
\frac{\partial Q^*}{\partial c_o} &= -\frac{a_2}{2\beta\theta^2} - \frac{(1-\tau)\gamma}{\theta}\frac{\partial q_{1n}}{\partial c_o} < 0,\\
\frac{\partial q_{2r}^*}{\partial c_o} &= \gamma\frac{\partial q_{1n}^*}{\partial c_o} > 0,\\
\frac{\partial q_{2n}^*}{\partial c_o} &= \theta\frac{\partial Q^*}{\partial c_o} - \tau\gamma\frac{\partial q_{1n}^*}{\partial c_o} = -\frac{a_2}{2\beta\theta} - \gamma\frac{\partial q_{1n}^*}{\partial c_o} < 0,\\
\frac{\partial q_2^*}{\partial c_o} &= -\frac{a_2}{2\beta\theta} < 0.
\end{aligned}$$

Thus, q_{1n}^*, q_{2r}^* increase in c_o and Q^*, q_{2n}^*, q_2^* decrease in c_o.

Case 2

$$\begin{aligned}
\frac{\partial Q^*}{\partial c_o} &= \frac{\partial q_{1n}^*}{\partial c_o} = -\frac{1}{2/a_1 + 2\beta[(1-\tau)\gamma+\theta]^2/a_2} < 0,\\
\frac{\partial q_{2r}^*}{\partial c_o} &= \gamma\frac{\partial q_{1n}^*}{\partial c_o} < 0, \frac{\partial q_{2n}^*}{\partial c_o} = (\theta - \tau\gamma)\frac{\partial q_{1n}^*}{\partial c_o} < 0,\\
\frac{\partial q_2^*}{\partial c_o} &= [\theta + (1-\tau)\gamma]\frac{\partial q_{1n}^*}{\partial c_o} < 0.
\end{aligned}$$

Thus, q_{1n}^*, q_{2r}^*, Q^*, q_{2n}^* and q_2^* decrease in c_o.

Case 4-6

$$\begin{aligned}
\frac{\partial Q^*}{\partial c_o} &= \frac{\partial q_{1n}^*}{\partial c_o} = -\frac{a_1}{2} < 0, \frac{\partial q_{2r}^*}{\partial c_o} = \gamma\frac{\partial q_{1n}^*}{\partial c_o} < 0,\\
\frac{\partial q_{2n}^*}{\partial c_o} &= -\gamma\frac{\partial q_{1n}^*}{\partial c_o} > 0, \frac{\partial q_2^*}{\partial c_o} = 0.
\end{aligned}$$

Thus, q_{1n}^*, Q^*, q_{2r}^* decrease in c_o, q_{2n}^* increases in c_o and q_2^* is constant with respect to c_o.

Case 7

$$\begin{aligned}
\frac{\partial Q^*}{\partial c_o} &= \frac{\partial q_{1n}^*}{\partial c_o} = -\frac{1}{2/a_1 + 2\beta\gamma^2/a_2} < 0,\\
\frac{\partial q_2^*}{\partial c_o} &= \frac{\partial q_{2r}^*}{\partial c_o} = \gamma\frac{\partial q_{1n}^*}{\partial c_o} < 0, \frac{\partial q_{2n}^*}{\partial c_o} = 0.
\end{aligned}$$

Thus, q_{1n}^*, q_{2r}^*, Q^* and q_2^* decrease in c_o.

Case 8

$$\frac{\partial Q^*}{\partial c_o} = \frac{\partial q_{1n}^*}{\partial c_o} = -\frac{a_1}{2} < 0.$$

Thus, q_{1n}^* and Q^* decrease in c_o and q_{2r}^*, q_{2n}^*, q_2^* is constant with respect to c_o.

Appendix 3.D.4 Sensitivity analysis with respect to τ

Case 1

$$\begin{aligned}
\frac{\partial q_{1n}^*}{\partial \tau} &= -\frac{a_1 c_o \gamma}{\theta} < 0, \frac{\partial q_{2r}^*}{\partial \tau} = \gamma \frac{\partial q_{1n}^*}{\partial \tau} < 0, \\
\frac{\partial Q^*}{\partial \tau} &= -\frac{\gamma}{\theta}\left(-q_{1n}^* + (1-\tau)\frac{\partial q_{1n}^*}{\partial \tau}\right) > 0, \\
\frac{\partial q_{2n}^*}{\partial \tau} &= \theta \frac{\partial Q^*}{\partial \tau} - \gamma q_{1n}^* - \tau\gamma \frac{\partial q_{1n}^*}{\partial \tau} \\
&= -\gamma\left(-q_{1n}^* + (1-\tau)\frac{\partial q_{1n}^*}{\partial \tau}\right) - \gamma q_{1n}^* - \tau\gamma\frac{\partial q_{1n}^*}{\partial \tau} = -\gamma \frac{\partial q_{1n}^*}{\partial \tau} > 0, \\
\frac{\partial q_2^*}{\partial \tau} &= \frac{\partial q_{2n}^*}{\partial \tau} + \frac{\partial q_{2r}^*}{\partial \tau} = 0.
\end{aligned}$$

Thus, as τ increases, q_{1n}^* and q_{2r}^* decrease and Q^*, q_{2n}^* increase and q_2 does not change.

Case 2

$$\begin{aligned}
\frac{\partial q_{1n}^*}{\partial \tau} &= \frac{\partial Q^*}{\partial \tau} = \frac{\frac{\beta^2\gamma S^2}{a_2^2} + \frac{2\beta\gamma KS}{a_1 a_2} - \frac{\beta\gamma I}{a_1 a_2}}{2(1/a_1 + \beta S^2/a_2)^2}, \\
\frac{\partial q_{2r}^*}{\partial \tau} &= \gamma \frac{\partial q_{1n}^*}{\partial \tau}.
\end{aligned}$$

where $S = ((1-\tau)\gamma + \theta)$, $I = M_2 - a_2 c_n$, $K = M_1 - a_1\Delta_0$. Since the denominator of $\partial q_{1n}^*/\partial\tau$ is always positive, the sign of $\partial q_{1n}^*/\partial\tau$ is same as that of the numerator, which is a quadratic equation with respect to S. Define N as the numerator of $\partial q_{1n}^*/\partial\tau$, i.e., $N = \frac{\beta^2\gamma S^2}{a_2^2} + \frac{2\beta\gamma KS}{a_1 a_2} - \frac{\beta\gamma I}{a_1 a_2}$. Non-negativity of S implies its only root to be

$$S' = \frac{\left(a_2\sqrt{\frac{1}{a_2}\left(K^2 a_2 + \beta I a_1\right)} - K a_2\right)}{\beta a_1}.$$

The upper and lower bounds for S namely S_u and S_l, can be derived from the optimality condition for case 2, which is $SK < I \leq SK + \frac{c_o}{\beta\theta}(S^2 a_1 \beta + a_2)$. Thus, if S satisfies the condition $S_l \leq S \leq S'$, then q_{1n}^*, q_{2r}^* and Q^* increase in τ; and if S satisfies the condition $S' \leq S \leq S_u$,then q_{1n}^*, q_{2r}^* and Q^* decrease in τ. Next, we continue with the sensitivity analysis for period 2.

$$\begin{aligned}
\frac{\partial q_{2n}^*}{\partial \tau} &= (\theta - \tau\gamma)\frac{\partial q_{1n}^*}{\partial \tau} - \gamma q_{1n}^*, \\
\frac{\partial q_2^*}{\partial \tau} &= ((1-\tau)\gamma + \theta)\frac{\partial q_{1n}^*}{\partial \tau} - \gamma q_{1n}^*.
\end{aligned}$$

However, due to the complexity of these expressions, we have not been able to derive closed form expressions for their roots which are third degree polynomials for τ. Besides finding roots, it should also be checked whether they are non-negative and satisfy the optimality condition for case 2, which is also non-trivial.

Cases 4-8

All decision variables are constant in τ.

Appendix 3.D.5 Sensitivity analysis with respect to θ

Case 1

$$\frac{\partial q_{1n}^*}{\partial \theta} = -\frac{a_1 c_o (1-\tau)\gamma}{2\theta^2} < 0.$$

Thus, q_{1n} decreases in θ.

$$\begin{aligned}
\frac{\partial Q^*}{\partial \theta} &= -\frac{M_2 - a_2 c_n}{2\theta^2} + \frac{a_2 c_o}{\beta\theta^3} + \frac{(1-\tau)\gamma(M_1 - a_1(c_n - \beta\gamma(c_n - c_r))) + 2(1-\tau)^2\gamma^2 a_1 c_o}{2\theta^3} \\
&= \frac{2a_2 c_o + \beta(1-\tau)\gamma(M_1 - a_1(c_n - \beta\gamma(c_n - c_r))) + 2(1-\tau)^2\gamma^2 a_1 c_o - \beta\theta(M_2 - a_2 c_n)}{2\beta\theta^3}.
\end{aligned}$$

Let $\theta' = 2a_2 c_o + \beta(1-\tau)\gamma(M_1 - a_1(c_n - \beta\gamma(c_n - c_r))) + 2(1-\tau)^2\gamma^2 a_1 c_o / [\beta(M_2 - a_2 c_n)]$

since $2\beta\theta^3 > 0$ always holds $\frac{\partial Q}{\partial \theta} < 0$ if $\theta < \theta'$ and $\frac{\partial Q}{\partial \theta} > 0$ if $\theta > \theta'$.

$$\begin{aligned}
\frac{\partial q_{2r}^*}{\partial \theta} &= \gamma \frac{\partial q_{1n}^*}{\partial \theta} = -\frac{a_1 c_o (1-\tau)\gamma^2}{2\theta^2} < 0, \\
\frac{\partial q_{2n}^*}{\partial \theta} &= \theta \frac{\partial Q^*}{\partial \theta} - \tau\gamma \frac{\partial q_{1n}^*}{\partial \theta} \\
&= \frac{a_2 c_o}{\beta\theta^2} + \frac{a_1 c_o (1-\tau)\gamma^2}{2\theta^2} > 0, \\
\frac{\partial q_2^*}{\partial \theta} &= \frac{\partial q_{2n}^*}{\partial \theta} + \frac{\partial q_{2r}^*}{\partial \theta} = \frac{a_2 c_o}{\beta\theta^2} > 0.
\end{aligned}$$

Thus q_{2r}^* decreases in θ and q_{2n}^* and q_2^* increase in θ.

Case 2

$$\begin{aligned}
\frac{\partial Q^*}{\partial \theta} &= \frac{\partial q_{1n}^*}{\partial \theta} = \frac{\frac{2\beta(M_2 - a_2 c_n)}{a_2} - \frac{2\beta^2[(1-\tau)\gamma+\theta]^2(M_2 - a_2 c_n)}{a_2^2} - \frac{4\beta[(1-\tau)\gamma+\theta](M_1 - a_1\Delta_0)}{a_1}}{[2/a_1 + 2\beta[(1-\tau)\gamma+\theta]^2/a_2]^2}, \\
\frac{\partial q_{2r}^*}{\partial \theta} &= \gamma \frac{\partial q_{1n}^*}{\partial \theta}.
\end{aligned}$$

Let $I = (M_2 - a_2 c_n)$, $K = (M_1 - a_1\Delta_0)$ and $S = [(1-\tau)\gamma + \theta]$. The denominator of $\partial Q^*/\partial\theta$ is always positive, and hence the sign of $\partial Q^*/\partial\theta$ is same as that of the numerator which is a quadratic equation with respect to S. Let $N = \frac{2\beta I}{a_2} - \frac{2\beta^2 S^2 I}{a_2^2} - \frac{4S\beta K}{a_1}$. The only non-negative root is

$$S' = \frac{a_2^2}{I\beta a_1}\left(\sqrt{\frac{1}{a_2^3}\left(K^2 a_2^3 + I^2\beta a_1^2\right)} - K\right).$$

The upper and lower bounds for S namely S_u and S_l, can be derived from the optimality condition for case 2, which is $SK < I \leq SK + \frac{c_o}{\beta\theta}(S^2 a_1\beta + a_2)$. Thus, if S satisfies the condition $S_l \leq S \leq S'$, then q_{1n}^*, q_{2r}^* and Q^* decrease in θ;and if S satisfies the condition $S' \leq S \leq S_u$, then q_{1n}^*, q_{2r}^* and Q^* increase in θ. Next, we continue with the sensitivity analysis for period 2

$$\begin{aligned}
\frac{\partial q_{2n}^*}{\partial\theta} &= (\theta - \tau\gamma)\frac{\partial q_{1n}^*}{\partial\theta} + q_{1n}^*, \\
\frac{\partial q_2^*}{\partial\theta} &= [\theta + (1-\tau)\gamma]\frac{\partial q_{1n}^*}{\partial\theta} + q_{1n}^*.
\end{aligned}$$

As in sensitivity analysis with respect to τ in case 2, these expressions are very complex and we have not been able to find the roots that are non-negative and satisfy the optimality condition for this case.

Case 4-8 The optimal solution does not depend on θ.

Appendix 3.D.6 Sensitivity analysis with respect to γ

Case 1

$$\begin{aligned}
\frac{\partial q_{1n}^*}{\partial\gamma} &= \frac{a_1}{2}(\frac{c_o(1-\tau)}{\theta} + \beta(c_n - c_r)) > 0, \\
\frac{\partial Q^*}{\partial\gamma} &= -\frac{(1-\tau)}{\theta}(q_{1n} + \gamma\frac{\partial q_{1n}}{\partial\gamma}) < 0, \\
\frac{\partial q_{2r}^*}{\partial\gamma} &= q_{1n}^* + \gamma\frac{\partial q_{1n}^*}{\partial\gamma} > 0, \\
\frac{\partial q_{2n}^*}{\partial\gamma} &= \theta\frac{\partial Q^*}{\partial\gamma} - \tau q_{1n}^* - \tau\gamma\frac{\partial q_{1n}^*}{\partial\gamma} < 0.
\end{aligned}$$

Thus, as γ increases, q_{1n}^*, q_{2r}^* increase and Q^*, q_{2n}^* decrease.

$$\begin{aligned}
q_2^* &= q_{2n}^* + q_{2r}^* = \theta Q^* + (1-\tau)\gamma q_{1n}^*, \\
\frac{\partial q_2^*}{\partial\gamma} &= \theta\frac{\partial Q^*}{\partial\gamma} + (1-\tau)q_{1n}^* + (1-\tau)\gamma\frac{\partial q_{1n}^*}{\partial\gamma} \\
&= -(1-\tau)q_{1n}^* - (1-\tau)\gamma\frac{\partial q_{1n}^*}{\partial\gamma} + (1-\tau)q_{1n}^* + (1-\tau)\gamma\frac{\partial q_{1n}^*}{\partial\gamma} = 0.
\end{aligned}$$

Thus q_2^* is constant with respect to γ.

Case 2

$$\begin{aligned}
\frac{\partial q_{1n}^*}{\partial\gamma} &= \frac{\partial Q^*}{\partial\gamma} = \frac{\begin{array}{c}\{[\beta(c_n - c_r) + \beta(1-\tau)\frac{M_2 - a_2 c_n}{a_2}][1/a_1 + \beta[(1-\tau)\gamma + \theta]^2/a_2] \\ -2(1-\tau)\frac{\beta[(1-\tau)\gamma+\theta]}{a_2}[\frac{M_1 - a_1\Delta_0}{a_1} + \beta((1-\tau)\gamma + \theta)\frac{M_2 - a_2 c_n}{a_2}]\}\end{array}}{2(1/a_1 + \beta((1-\tau)\gamma + \theta)^2/a_2)^2}, \\
\frac{\partial q_{2n}^*}{\partial\gamma} &= (\theta - \tau\gamma)\frac{\partial q_{1n}^*}{\partial\gamma} - \tau q_{1n}^*.
\end{aligned}$$

Since the denominator of $\partial q_{1n}^*/\partial\gamma$ is always positive the sign of $\partial q_{1n}^*/\partial\gamma$ is same as the numerator that is a quadratic equation with respect to γ.

Denote $K = M_1 - a_1\Delta_0$, $S = [\theta + (1-\tau)\gamma]$, $I = M_2 - a_2 c_n$. Then,

$$\begin{aligned}
\frac{\partial q_{2r}^*}{\partial\gamma} &= q_{1n}^* + \gamma\frac{\partial q_{1n}^*}{\partial\gamma} \\
&= \frac{\left\{\begin{array}{c} \left(\frac{K}{a_1} + \frac{\beta IS}{a_2}\right)\left(\frac{1}{a_1} + \frac{\beta S^2}{a_2}\right) + \left(\gamma\beta(c_n - c_r) + \frac{\gamma\beta(1-\tau)I}{a_2}\right) \\ \left(\frac{1}{a_1} + \frac{\beta S^2}{a_2}\right) - \frac{2(1-\tau)\beta\gamma S}{a_2}\left(\frac{K}{a_1} + \frac{\beta IS}{a_2}\right) \end{array}\right\}}{2(1/a_1 + \beta((1-\tau)\gamma + \theta)^2/a_2)^2},
\end{aligned}$$

$$\begin{aligned}
\frac{\partial q_2^*}{\partial\gamma} &= [\theta + (1-\tau)\gamma]\frac{\partial q_{1n}^*}{\partial\gamma} + (1-\tau)q_{1n}^* \\
&= \frac{\left\{\begin{array}{c} \left(\frac{K(1-\tau)}{a_1} + \frac{(1-\tau)\beta IS}{a_2}\right)\left(\frac{1}{a_1} + \frac{\beta S^2}{a_2}\right) + \frac{S\beta(c_n - c_r) + S\beta(1-\tau)I}{a_2} \\ \left(\frac{1}{a_1} + \frac{\beta S^2}{a_2}\right) - \frac{2(1-\tau)\beta S^2}{a_2}\left(\frac{K}{a_1} + \frac{\beta IS}{a_2}\right) \end{array}\right\}}{2(1/a_1 + \beta((1-\tau)\gamma + \theta)^2/a_2)^2} \\
&= \frac{\frac{K(1-\tau)}{a_1^2} + \frac{2(1-\tau)\beta IS}{a_1 a_2} - \frac{2K(1-\tau)\beta S^2}{a_1 a_2} + \frac{(c_n - c_r)\beta S}{a_1 a_2} + \frac{(c_n - c_r)\beta^2 S^2}{a_2^2}}{2(1/a_1 + \beta((1-\tau)\gamma + \theta)^2/a_2)^2} > 0.
\end{aligned}$$

Since $SK < I$ holds for case 2, q_{2r}^* and q_2^* are increasing in γ.

Cases 4-6

$$\begin{aligned}
\frac{\partial q_{1n}^*}{\partial\gamma} &= \frac{\partial Q^*}{\partial\gamma} = \frac{a_1}{2}\beta(c_n - c_r) > 0, \\
\frac{\partial q_{2r}^*}{\partial\gamma} &= q_{1n}^* + \gamma\frac{\partial q_{1n}^*}{\partial\gamma} > 0, \\
\frac{\partial q_{2n}^*}{\partial\gamma} &= -\frac{\partial q_{2r}^*}{\partial\gamma} < 0, \\
q_2^* &= q_{2n}^* + q_{2r}^* = I_n \Longrightarrow \frac{\partial q_2^*}{\partial\gamma} = 0.
\end{aligned}$$

Thus, as γ increases, q_{1n}^*, q_{2r}^* and Q^* increase, q_{2n}^* decrease and q_2^* remains constant.

Case 7

$$\frac{\partial q_{1n}^*}{\partial\gamma} = \frac{\partial Q^*}{\partial\gamma} = \frac{\frac{\beta(M_2 - a_2 c_n)}{a_1 a_2} - \frac{\beta^2\gamma^2(M_2 - a_2 c_r)}{a_2^2} - \frac{2\beta\gamma(M_1 - a_1(c_n + c_o))}{a_1 a_2}}{2(1/a_1 + \beta\gamma^2/a_2)^2}.$$

Since the denominator is always positive, the sign of the above expression is equal to that of the numerator, which is a quadratic equation of γ. Let $N = -\frac{\beta^2(M_2 - a_2 c_r)\gamma^2}{a_2^2} - \frac{2\beta(M_1 - a_1(c_n + c_o))\gamma}{a_1 a_2} + \frac{\beta(M_2 - a_2 c_r)}{a_1 a_2}$, $A = -\frac{\beta(M_2 - a_2 c_r)}{a_2^2}$, $B = -\frac{2\beta(M_1 - a_1(c_n + c_o))}{a_1 a_2}$, $C = \frac{\beta(M_2 - a_2 c_r)}{a_1 a_2}$. Since γ is non-negative the only root of the numerator is

$$\gamma' = \frac{a_2\left(\sqrt{\frac{(M_1-a_1(c_n+c_o))^2}{a_1^2}+\frac{\beta(M_2-a_2c_r)^2}{a_1a_2}}\right)-\frac{M_1-a_1(c_n+c_o)}{a_1}}{\beta(M_2-a_2c_r)}.$$

Moreover, we can determine the upper and lower limits for γ namely, γ_u and γ_l from the optimality condition of case 2 that is presented in Table 3. To find γ_l we have to consider the optimality condition $M_2 - a_2c_n < \gamma(M_1 - a_1\Delta_0)$, which can be rewritten as a quadratic equation in γ. Since γ is non-negative, the only root of the equation is

$$\gamma_l = \frac{\sqrt{[M_1 - a_1(c_n + c_o)]^2 + 4\beta a_1(c_n - c_r)(M_2 - a_2c_n)} - [M_1 - a_1(c_n + c_o)]}{2\beta a_1(c_n - c_r)}.$$

To find γ_u we have to consider the condition $M_2 - a_2c_r \geq \gamma[M_1 - a_1(c_n - c_r)]$. Then the upper bound is

$$\gamma_u = \frac{M_2 - a_2c_r}{M_1 - a_1(c_n + c_o)}.$$

So, q_{1n} decreases in γ if $\gamma_l < \gamma \leq \gamma'$, and q_{1n} increases in γ if $\gamma' \leq \gamma \leq \gamma_u$. We also have

$$\begin{aligned}
\frac{\partial q_{2r}^*}{\partial \gamma} &= \frac{\partial q_2^*}{\partial \gamma} = q_{1n}^* + \gamma\frac{\partial q_{1n}^*}{\partial \gamma} \\
&= \frac{\frac{2\beta\gamma(M_2-a_2c_r)}{a_1a_2} + \frac{M_1-a_1(c_n+c_o)}{a_1^2} - \frac{\beta\gamma^2(M_1-a_1(c_n+c_o)}{a_1a_2}}{2(1/a_1 + \beta\gamma^2/a_2)^2} > 0,
\end{aligned}$$

since $M_2 - a_2c_r \geq \gamma[M_1 - a_1(c_n + c_o)]$ holds. Thus, q_{2r}^* and q_2^* increase in γ.

Case 8

The optimal solution does not depend on γ.

Appendix 3.D.7 Sensitivity analysis with respect to M_1

Case 1

$$\begin{aligned}
\frac{\partial q_{1n}^*}{\partial M_1} &= \frac{1}{2} > 0, \\
\frac{\partial Q^*}{\partial M_1} &= -\frac{(1-\tau)\gamma}{2\theta} < 0, \\
\frac{\partial q_{2r}^*}{\partial M_1} &= \gamma\frac{\partial q_{1n}^*}{\partial M_1} = \frac{\gamma}{2} > 0, \\
\frac{\partial q_{2n}^*}{\partial M_1} &= \theta\frac{\partial Q^*}{\partial M_1} - \tau\gamma\frac{\partial q_{1n}^*}{\partial M_1} = -\frac{\gamma}{2} < 0, \\
\frac{\partial q_2^*}{\partial M_1} &= \frac{\partial q_{2n}^*}{\partial M_1} + \frac{\partial q_{2r}^*}{\partial M_1} = 0.
\end{aligned}$$

Thus, q_{1n}^* and q_{2r}^* increase in M_1, $Q^*$and q_{2n}^* decrease in M_1 and q_2^* does not depend on M_1.

Case 2

$$\begin{aligned}
\frac{\partial Q^*}{\partial M_1} &= \frac{\partial q_{1n}^*}{\partial M_1} = \frac{1/a_1}{2/a_1 + 2\beta[(1-\tau)\gamma+\theta]^2/a_2} > 0, \\
\frac{\partial q_{2r}^*}{\partial M_1} &= \gamma\frac{\partial q_{1n}^*}{\partial M_1} > 0, \frac{\partial q_{2n}^*}{\partial M_1} = (\theta - \tau\gamma)\frac{\partial q_{1n}^*}{\partial M_1} > 0, \\
\frac{\partial q_2^*}{\partial M_1} &= [\theta + (1-\tau)\gamma]\frac{\partial q_{1n}^*}{\partial M_1} > 0.
\end{aligned}$$

Thus, as M_1 increases, q_{1n}^*, Q^*, q_{2n}^*, q_{2r}^*, q_2^* increase.

Case 4-6

$$\begin{aligned}
\frac{\partial Q^*}{\partial M_1} &= \frac{\partial q_{1n}^*}{\partial M_1} = \frac{1}{2} > 0, \frac{\partial q_{2r}^*}{\partial M_1} = \gamma\frac{\partial q_{1n}^*}{\partial M_1} = \frac{\gamma}{2} > 0, \\
\frac{\partial q_{2n}^*}{\partial M_1} &= -\frac{\gamma}{2} < 0, \frac{\partial q_2^*}{\partial M_1} = \frac{\partial q_{2r}^*}{\partial M_1} +, \frac{\partial q_{2n}^*}{\partial M_1} = 0.
\end{aligned}$$

Thus, as M_1 increases, q_{1n}^*, Q^*,and q_{2r}^* increase, q_{2n}^* decrease and q_2^* does not change

Case 7

$$\begin{aligned}
\frac{\partial Q^*}{\partial M_1} &= \frac{\partial q_{1n}^*}{\partial M_1} = -\frac{1}{2/a_1 + 2\beta\gamma^2/a_2} < 0, \\
\frac{\partial q_2^*}{\partial M_1} &= \frac{\partial q_{2r}^*}{\partial M_1} = \gamma\frac{\partial q_{1n}^*}{\partial M_1} < 0, \frac{\partial q_{2n}^*}{\partial M_1} = 0.
\end{aligned}$$

Thus, as M_1 increases, q_{1n}^*, Q^*, q_{2r}^* and q_2^* decrease.

Case 8

$$\frac{\partial Q^*}{\partial M_1} = \frac{\partial q_{1n}^*}{\partial M_1} = \frac{1}{2} > 0.$$

Thus, as M_1 increases, Q^* and q_{1n}^* increase and q_{2n}^*, q_{2r}^*, q_2^* do not change.

Appendix 3.D.8 Sensitivity analysis with respect to M_2

Case 1

$$\begin{aligned}
\frac{\partial q_{1n}^*}{\partial M_2} &= 0, \frac{\partial Q^*}{\partial M_2} = \frac{1}{2\theta} > 0, \\
\frac{\partial q_{2r}^*}{\partial M_2} &= \gamma\frac{\partial q_{1n}^*}{\partial M_2} = 0, \frac{\partial q_{2n}^*}{\partial M_2} = \frac{1}{2} > 0, \\
\frac{\partial q_2^*}{\partial M_2} &= \frac{1}{2} > 0.
\end{aligned}$$

Thus, as M_2 increases, Q^*, q_{2n}^* and q_2^* increase and q_{1n}^* and q_{2r}^* do not change.

Case 2

$$\begin{aligned}
\frac{\partial Q^*}{\partial M_2} &= \frac{\partial q_{1n}^*}{\partial M_2} = \frac{\beta[(1-\tau)\gamma+\theta]}{2a_2(1/a_1+\beta[(1-\tau)\gamma+\theta]^2/a_2)} > 0, \\
\frac{\partial q_{2r}^*}{\partial M_2} &= \gamma\frac{\partial q_{1n}^*}{\partial M_2} > 0, \frac{\partial q_{2n}^*}{\partial M_2} = (\theta-\tau\gamma)\frac{\partial q_{1n}^*}{\partial M_2} > 0, \\
\frac{\partial q_2^*}{\partial M_2} &= [\theta+(1-\tau)\gamma]\frac{\partial q_{1n}^*}{\partial M_2} > 0.
\end{aligned}$$

Thus, as M_2 increases, q_{1n}^*, Q, q_{2n}^*, q_{2r}^*, q_2^* increase.

Case 4-6

$$\begin{aligned}
\frac{\partial Q^*}{\partial M_2} &= \frac{\partial q_{1n}^*}{\partial M_2} = \frac{\partial q_{2r}^*}{\partial M_2} = 0, \\
\frac{\partial q_{2n}^*}{\partial M_2} &= \frac{1}{2} > 0, \frac{\partial q_2^*}{\partial M_2} = \frac{\partial q_{2r}^*}{\partial M_2} + \frac{\partial q_{2n}^*}{\partial M_2} = \frac{1}{2} > 0.
\end{aligned}$$

Thus, Q^*, q_{1n}^* and q_{2r}^* are independent of M_2, and q_{2n}^* and q_2^* increase as M_2 increases.

Case 7

$$\begin{aligned}
\frac{\partial Q^*}{\partial M_2} &= \frac{\partial q_{1n}^*}{\partial M_2} = \frac{\beta\gamma}{2a_2(1/a_1+\beta\gamma^2/a_2)} > 0, \\
\frac{\partial q_2^*}{\partial M_2} &= \frac{\partial q_{2r}^*}{\partial M_2} = \gamma\frac{\partial q_{1n}^*}{\partial M_2} > 0, \frac{\partial q_{2n}^*}{\partial M_2} = 0.
\end{aligned}$$

Thus, as M_2 increases, q_{1n}^*, Q^*, q_{2r}^*, q_2^* increase.

Case 8

$$\begin{aligned}
\frac{\partial q_{2r}^*}{\partial M_2} &= \frac{\partial q_2^*}{\partial M_2} = \frac{1}{2} > 0, \\
\frac{\partial Q^*}{\partial M_2} &= \frac{\partial q_{1n}^*}{\partial M_2} = \frac{\partial q_{2n}^*}{\partial M_2} = 0.
\end{aligned}$$

Thus, as M_2 increases, q_{2r}^*, q_2^* increase and q_{1n}^*, Q^* do not change.

Chapter 4

Competition for cores in remanufacturing

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In this chapter, we study competition between an original equipment manufacturer (OEM) and an independently operating remanufacturer (IO). Different from the existing literature, the OEM and IO compete not only for selling their products but also for collecting returned products (cores) through their acquisition prices. We consider a two-period model with manufacturing by the OEM in the first period, and manufacturing as well as remanufacturing in the second period. We find the optimal policies for both players by establishing a Nash Equilibrium in the second period, and then determine the optimal manufacturing decision for the OEM in the first period. This leads to a number of managerial insights. One interesting result is that the acquisition price of the OEM only depends on its own cost structure, and not on the acquisition price of the IO. Further insights are obtained from a numerical investigation. We find that when the cost benefits of remanufacturing diminishes and the IO has more chance to collect the available cores, the OEM manufactures less in the first period as the market in the second period gets larger to protect its market share. Finally, we consider the case where consumers have lower willingness to pay for the remanufactured products and find that in that case remanufacturing becomes less

profitable overall.

4.1 Introduction

The importance of sustainable and environmental processes has increased over the last fifty years, as industrialization and population growth have increasingly burdened the environment. To reduce this burden, European companies have been made legally responsible for collecting their end-of-life products and adopting the sustainable production strategy of product recovery (recycling and remanufacturing). In other countries where firms are not legally bound to collect used-products, many still do so because of the economic benefits (Kaya, 2010).

Remanufacturing is defined as the process of bringing used products to a like-new functional state with warranty to match. The steps of a remanufacturing process are: collecting the used/discarded items, disassembling those items into their parts, cleaning and inspecting each part, and finally reassembling the parts and testing the remanufactured products to function as if they were new.

Remanufacturing has numerous benefits for the original equipment manufacturers (OEMs). These include savings in labor, material and energy costs; shorter production lead times; balanced production lines; new market development opportunities, and a positive, socially concerned image (McConocha and Speh, 1991). Adopting remanufacturing as a part of production strategy allowed Caterpillar to create a new market among contractors who cannot afford to purchase a Caterpillar product outright (Gutowski et al., 2001). In the 2008/2009 financial year, Fuji Xerox Australia remanufactured more than 230,000 equipment parts, equating to a $6 million cost-saving compared to sourcing new parts (www.fujixerox.com).

There is also an encouraging market for remanufactured products in the US. According to U.S International Trade Commission, production of remanufactured products in U.S increased from $37.3 billion in 2009 to $43.0 billion in 2011 (www.usitc.gov).

Despite of the numerous benefits, many firms have reservations about adopting a remanufacturing strategy related to concerns about internal cannibalization and external competition. Starting with the first, when consumer populations for new and remanufactured products overlap, selling remanufactured products may reduce new product sales (even if different marketing channels are used). Firms may consider this situation undesirable as it can cut revenues, even if the profit margin of a new product is less than that of a remanufactured product. Internal cannibalization is not a concern for an OEM if new and remanufactured products are perceived as the same to customers (Atasu et al., 2010), such as refillable containers and single-use cameras. Also, in Japan Fuji Xerox incorporated reused components in new products. Thus, all products may include reused components and there is no distinction between new and remanufactured products (Matsumoto and Umeda, 2011).

External competition is the other major issue for firms. If an OEM does not offer a remanufactured version of a popular product (because of cannibalization concerns), independent operators (IOs) may enter the market and compete for returns and a share of the market. In this case, the OEM faces external competition rather than internal cannibalization, or both if the OEM itself also remanufactures products. Guide (2000) estimates that 95% of the remanufacturing programs are not managed by the original producers. In the mobile phone and automobile parts remanufacturing industry, for instance, independent operators have a clear dominance. Kodak also competes with IOs today to collect and remanufacture the single-used cameras. Moreover, as Atasu et al. (2010) state, it is very difficult for OEMs to enter the remanufacturing market after it becomes dominated by IOs. Thus, remanufacturing decisions and the positioning (pricing) of remanufactured products are crucial for OEMs. However, there is a lack of analytical tools for guiding firms in those decisions. As Ferguson and Toktay (2006) and Atasu et al. (2008) state, most of the firms today do not have a clear understanding of how to adopt remanufacturing and position a remanufactured product.

In this chapter, we analyze the effect of competition between one OEM and one IO on the optimal production and pricing strategies. Our aim is to find the optimal strategies for the agents in the case of competition and to determine the effect of competition on the remanufacturing strategy. We determine the optimal manufacturing and remanufacturing decisions for an OEM and an IO in a two-period setting.

The rest of the chapter is organized as follows: The next section reviews the closely related literature and explains our contribution. Section 4.3 describes the model in detail and in Section 4.4 we characterize the optimal policy for periods 2 and 1. A numerical study is conducted in Section 4.5 to understand the effect of each parameter on the optimal solution and to gain insights into the effect of remanufacturing on the total production in both periods and on the OEM's profit. In Section 4.6, we consider the case where new and remanufactured products are sold at different prices and conduct a numerical study similar to that in Section 4.5 to understand how different selling price for products change the effect of parameters on the optimal solution. Finally, in Section 4.7, a brief summary of the findings and managerial insights is provided, and avenues for further research are discussed.

4.2 Closely related literature

In this section, we review the literature and point out key differences with our study.

There are numerous studies on closed-loop supply chains and remanufacturing in the current literature. Fleischmann et al. (1997) provide a review for early studies and Guide and Van Wassenhove (2009) describe the evolution of the research on closed-loop supply chains. Souza (2013) provides a review and a tutorial of the literature on closed loop supply chains. He classified the literature in terms of strategic, tactical, and operational issues. Tang and Zhou (2012) provide a review of recent studies (between 1995-2012) in closed loop supply chain literature based on a PPP (planet, people and profit) framework. Another recent survey is on production planning and control for remanufacturing which is conducted by Junior and Filho (2012).

Competition in remanufacturing is a growing branch in the remanufacturing literature. Heese et al. (2005) study a model of two OEMs and determine the conditions for the OEMs to sell remanufactured products along with original products, also taking product substitutability into account. Atasu et al. (2008) also consider direct OEM competition and include market growth in their two-period setting. They assume that cores can be collected at a fixed price, but in our study we assume that acquisition prices for used products are decision variables.

Ferguson and Toktay (2006) consider competition between a single OEM and a single IO in a two-period model. They assume that an IO will only consider entering the market if the OEM decides not to remanufacture itself. They argue that this is reasonable when the OEM enjoys a brand advantage for its remanufactured product over the entrant's product or if the OEM enjoys a first-mover advantage in the recovery of the cores needed for remanufacturing. Majumder and Groeneveld (2001) also consider the competition between a single OEM and a single IO in a two-period setting, and assume that the shell allocation mechanism is exogenous, i.e., the fractions of returns to the OEM and IO are fixed. Ferrer and Swaminatham (2006) extend the model of Majumder and Groenevelt (2001) to a multi-period setting. Different from these contributions, in our model, the OEM and 3rd party remanufacturer compete with their acquisition prices for cores.

Webster and Mitra (2007) and Mitra and Webster (2008) also analyze a two-period game theoretic model that captures the competition between a single OEM and a single IO. Building on the previous two-period model studies, both studies examine the effect of legislations and regulations on the remanufacturing activities. Webster and Mitra (2007) study the effect of take-back laws on the remanufacturing activities by considering two different implementations of take back laws, namely, (i) the OEM has the control over the returns sold to IO and (ii) the OEM has no control over the returns sold to IO. Mitra and Webster (2008) analyze the effect of government subsidies on remanufacturing activities. In both studies it is assumed that the OEM does not take part in remanufacturing activities.

Debo et al. (2005) investigate joint technology selection and pricing decisions for new and remanufactured products. They derive the manufacturer's optimal remanufacturing decisions as well as conditions on the viability of remanufacturing. They also extend their results to the case of competing remanufacturers. They assume that the price of used remanufacturable products depends only on the supply of such products. So, different from our model, all parties offer the same acquisition price, and all available (remanufacturable) cores are acquired.

4.3 System description

In this chapter, we consider the competition between an original-equipment manufacturer (OEM) and an independent operator (IO) in a two-period setting. We model the decision process as follows. At the beginning of period 1, the OEM decides on the number of products to manufacture, q_{1n}. Returned products are not yet available in that period. At the start of the second period, both the OEM and the IO decide on the acquisition prices for returned products, s_o and s_i, respectively. After obtaining the returned products, both decide on the number of returns to remanufacture in period 2, namely q_{2o} and q_{2i}. At this point the OEM also decides on the number of products to manufacture, q_{2n}. Obviously, the number of remanufactured products by the OEM and IO are restricted by the number of products returned to them, r_o and r_i respectively.

For modeling the demand functions, we adopt the same utility-based approach as Debo et al. (2005) and Ferguson and Toktay (2006) where each consumer uses at most one product and has willingness to pay for that product, θ, which is uniformly distributed between 0 and 1. In the first period, only new products are offered with selling price p_1 and the utility of a consumer for buying a new product is $U_1 = \theta - p_1$. If the market size of the first period is M_1, the quantity sold in that period, q_{1n} becomes $q_{1n} = M_1(1 - p_1)$. Then, letting $\delta_1 = 1/M_1$, we can derive the inverse demand function as follows,

$$p_1 = \delta_1(M_1 - q_{1n}). \tag{4.1}$$

In the second period, both manufactured and remanufactured products are sold. In the main part of this study, we assume that consumers cannot distinguish between the new and remanufactured products i.e., consumers view the remanufactured products as perfect substitutes of new ones. As stated before, in practice, this holds for some products (such as single-use cameras, refillable cylinders and Fuji Xerox products in Japan), but obviously not for all products. Although in the main body of the study we consider indistinguishable new and remanufactured products, in Section 4.6, we relax this assumption and consider different selling price for the new and remanufactured products.

Using the same argument above, the selling price for this period, p_2 is derived in terms of the amount of new products offered by the OEM, q_{2n} and remanufactured products offered by the OEM q_{2o} and by the IO, q_{2i}, as

$$p_2 = \delta_2(M_2 - q_{2n} - q_{2o} - q_{2i}), \tag{4.2}$$

where M_2 is the market size in period 2, and $\delta_2 = 1/M_2$.

To make the problem non-trivial and allow profitable manufacturing and remanufacturing for the OEM, we assume that the cost of manufacturing new products, c_n is less than $\delta_2 M_2$ and, greater than the remanufacturing cost for the OEM, c_o, i.e., $\delta_2 M_2 > c_n > c_o$ and $\delta_1 M_1 > c_n$.

We also assume that the number of products manufactured by the OEM in period 2 is positive. This will hold in most real-life settings, as the number of remanufacturable returns is typically insufficient to switch 'fully' from manufacturing to remanufacturing. We remark that the analysis for the case without manufacturing in the second period is similar to the one that we will present, but will be omitted as this case is unrealistic and not as insightful.

Competition for return is modelled by modifying the commonly used demand attraction model. This model can be derived axiomatically based on simple assumptions about consumer behavior (Luce, 1959). It is a popular model for modeling competition for demands and, as we will argue, is also suitable for modeling competition for returns in the following adapted form.

$$r_o = \beta q_{1n} \frac{\alpha_o s_o}{\alpha_o s_o + \alpha_i s_i + \gamma}, \tag{4.3}$$

$$r_i = \beta q_{1n} \frac{\alpha_i s_i}{\alpha_o s_o + \alpha_i s_i + \gamma}, \tag{4.4}$$

where $0 < \beta \leq 1, \alpha_o > 0, \alpha_i > 0, \gamma \geq 0$.

This generalizes the demand attraction model in two ways. First, the additional parameter β is introduced as it may not be possible to collect and remanufacture all items that were manufactured in the first period, even at very high acquisition prices (Debo et al., 2005 and Geyer et al., 2007). Second, α_o and α_i are positive rather than negative, as higher acquisition prices lead to more returns. We remark that, similar to the demand attraction model, our return attraction model can be generalized by using power functions instead of linear functions. However, as price-return sensitivity is already modelled via α_o, α_i and γ, and to keep the analysis tractable, we do not consider this generalized form.

A list of notation is given in Table 4.1. The objective of both the OEM and the IO is to maximize their total profit, which is calculated as the difference between the sales revenue and the acquisition and production costs. In the next section, we will determine the Nash equilibrium for these two players and show that it is unique.

4.4 Optimal policies

In this section, we determine the optimal policies for both players. We start in Section 4.4.1 by determining the unique, as it turns out to be, Nash equilibrium for the OEM and IO in period 2, given any value of q_{1n}. Furthermore, we obtain insights from an analytical sensitivity study. In Section 4.4.2, we show that the total profit over both periods for the OEM is concave in q_{1n}, and determine the first order optimality condition.

Model Parameters	
$M_k; k = 1, 2$	Potential market in period k
$\delta_k; k = 1, 2$	Constant that links selling price to sales in period k
c_n	OEM's cost of manufacturing per new product
c_o	OEM's cost of remanufacturing per remanufactured product
c_i	IO's cost of remanufacturing per remanufactured product
$\alpha_o, \alpha_i, \beta, \gamma$	Constants in the return attraction model
(Decision) Variables	
$p_k; k = 1, 2$	Sales price in period k
s_o	Acquisition price offered by the OEM (in period 2)
s_i	Acquisition price offered by the IO (in period 2)
$q_{kn}; k = 1, 2$	Number of new products manufactured by the OEM in period k
q_{2o}	Number of products remanufactured by the OEM (in period 2)
q_{2i}	Number of products remanufactured by the IO (in period 2)
r_o	Number of products returned to the OEM (in period 2)
r_i	Number of products returned to the IO (in period 2)
Other notation	
$\Pi_{ko}; k = 1, 2$	OEM's profit in period k
Π_o	OEM's total profit
Π_{2i}	IO's profit in period 2
Δ	$(a + b - 3\beta q_{1n})^2 + 12b\beta q_{1n}$
a	$M_2(1 + c_n - 2c_i)$
b	$4M_2(\alpha_o s_o^* + \gamma)/\alpha_i$
f'	First derivative of function f with respect to q_{1n}
f''	Second derivative of function f with respect to q_{1n}
*	Denotes optimality, e.g. s_o^* is the optimal acquisition price for the OEM

Table 4.1: Notation

4.4.1 Nash equilibrium for period 2

The OEM's profit in period 2 is

$$\Pi_{2o} = p_2(q_{2n} + q_{2o}) - c_n q_{2n} - c_o q_{2o} - s_o r_o. \tag{4.5}$$

The IO only operates in period 2 and IO's profit is given by

$$\Pi_i = \Pi_{2i} = p_2 q_{2i} - c_i q_{2i} - s_i r_i. \tag{4.6}$$

An important observation from (4.5) and (4.6) is that neither player has an incentive to acquire returns that it will not remanufacture, as it can increase its own profits by acquiring fewer returns at a higher price. Therefore, our search for Nash equilibria can be restricted to solutions where $q_{2o} = r_o$ and $q_{2i} = r_i$. Such solutions are completely characterized by the set of production quantities (q_{2n}, q_{2o}, q_{2i}). The corresponding acquisition and sales prices result from (4.2) to (4.4).

From (4.3) and (4.4), and by using $q_{2o} = r_o$ and $q_{2i} = r_i$, we get

$$\frac{\alpha_o s_o}{\alpha_i s_i} = \frac{q_{2o}}{q_{2i}}. \tag{4.7}$$

Using (4.2) - (4.4), it is easy to see that profit function (4.6) is concave in q_{2i}. Also, (4.5) is jointly concave in (q_{2n}, q_{2o}). (See Appendix 3.B.) Hence, there exists a unique Nash Equilibrium $(q_{2n}^*, q_{2o}^*, q_{2i}^*)$ given by the following first-order conditions:

$$\begin{aligned} M_2 - 2q_{2n}^* - 2q_{2o}^* - q_{2i}^* - c_n/\delta_2 &= 0, & (4.8)\\ M_2 - 2q_{2n}^* - 2q_{2o}^* - q_{2i}^* - c_o/\delta_2 - 2s_o^*/\delta_2 &= 0, & (4.9)\\ M_2 - q_{2n}^* - q_{2o}^* - 2q_{2i}^* - c_i/\delta_2 - 2s_i^*/\delta_2 &= 0. & (4.10) \end{aligned}$$

The unique Nash equilibrium that results from these conditions is given in Theorem 4.4.1.

Theorem 4.4.1 *The production quantities and acquisition prices for the unique Nash equilib-*

rium are given by

$$
\begin{aligned}
s_o^* &= \frac{c_n - c_o}{2}, && (4.11)\\
q_{2i}^* &= \frac{a + b + 3\beta q_{1n} - \Delta^{1/2}}{6}, && (4.12)\\
s_i^* &= \frac{q_{2i}^*(\alpha_o s_o^* + \gamma)}{\alpha_i(\beta q_{1n} - q_{2i}^*)}, && (4.13)\\
q_{2o}^* &= \frac{\alpha_o s_o^*}{\alpha_o s_o^* + \gamma}(\beta q_{1n} - q_{2i}^*), && (4.14)\\
q_{2n}^* &= \frac{1}{2}\left(M_2 - c_n/\delta_2 - 2\beta q_{1n}\frac{\alpha_o s_o^*}{\alpha_o s_o^* + \gamma}\right) + \frac{\alpha_o s_o^* - \gamma}{2(\alpha_o s_o^* + \gamma)}q_{2i}^*, && (4.15)
\end{aligned}
$$

Proof. See Appendix 4.B. ■

An interesting observation from Theorem 4.4.1 is that the acquisition price for cores offered by the OEM only depends on the cost difference $c_n - c_o$ between manufacturing and remanufacturing. So, the OEM's acquisition price is independent from that of the IO, and in this respect the OEM does not compete for cores. This result can be explained as follows. For any given demand structure, the marginal production cost must be equal to the marginal revenue (giving a marginal profit of zero) for the optimal solution. Therefore, as long as there is still production of new products in the second period, the total production (i.e., new and remanufactured) is not affected by the acquisition price and associated number of returns, since the marginal cost of producing one additional new product must remain unchanged. It is also obvious that the acquisition price must be less than $c_n - c_o$ for remanufacturing to be profitable for the OEM. However, the result that the optimal acquisition price is exactly half of that "maximum saving per return" results from our modeling of the competition for cores.

The complexity of the expressions in Theorem 4.4.1 for the other decision variables in period 2 does not allow us to directly obtain further insights into their dependency on q_{1n} and on the model parameters. Therefore, we obtain such insights analytically and numerically. We start by analyzing the impact of q_{1n} on the optimal decisions in period 2 in Theorem 4.4.2.

Theorem 4.4.2 *The sensitivity of the optimal decision variables for period 2 with respect to* q_{1n} *is as follows.*

(i) q_{2i}^* *is increasing in* q_{1n} *and* $0 < q_{2i}^{*\prime} < \beta$*;*

(ii) q_{2n}^* *is decreasing in* q_{1n} *and* q_{2o}^* *is increasing in* q_{1n}*;*

(iii) $(q_{2n}^* + q_{2o}^*)$ *is decreasing in* q_{1n}*;*

(iv) $(q_{2n}^* + q_{2o}^* + q_{2i}^*)$ *is increasing in* q_{1n}*;*

(v) s_o^* *is constant in* q_{1n}*;*

(vi) s_i^* *is decreasing in* q_{1n}*.*

Proof. See Appendix 4.B. ■

Results (i) and (ii) from Theorem 4.4.2 are intuitive. A larger number of available cores leads to more returns for both the OEM and the IO. To compensate for the increased number of remanufactured products in the market, the OEM manufactures less new products. And, as (iii) shows, the total production by the OEM in period 2 decreases. However, the OEM and IO together sell more products in period 2, as stated in (iv), and so the OEM loses market share. We have already discussed the independency of the OEM's acquisition price from q_{1n} (and the other decision variables), and (v) is included for completeness. Result (vi) shows that the IO exploits a larger availability of cores by offering a lower acquisition price.

So, we know from Theorem 4.4.2 that increased manufacturing in the first period leads to an increased amount of remanufacturing by both the OEM and the IO in the second period. Theorem 4.4.3 shows further that an increase in q_{1n} leads to an increased share of returns for the OEM, and to a reduction in remanufacturing relative to the number of available cores.

Theorem 4.4.3 $q_{2i}^*/(\beta q_{1n})$ *is decreasing in* q_{1n}*,* $q_{2o}^*/(\beta q_{1n})$ *is increasing in* q_{1n}*, and* $(q_{2i}^* + q_{2o}^*)/(\beta q_{1n})$ *is decreasing in* q_{1n}*.*

Proof. See Appendix 4.B. ■

We next examine the relationships between the acquisition prices s_i^* and s_o^* and between the remanufactured quantities q_{2i}^* and q_{2o}^*.

Theorem 4.4.4 *(a) We have*

$$s_i^* \begin{cases} > s_o^*, & q_{1n} < \hat{q}_{1n}; \\ = s_o^*, & q_{1n} = \hat{q}_{1n}; \\ < s_o^*, & q_{1n} > \hat{q}_{1n}, \end{cases} \tag{4.16}$$

where $\hat{q}_{1n}$ is uniquely determined by

$$\frac{q_{2i}^*(\alpha_o s_o^* + \gamma)}{\alpha_i(\beta\hat{q}_{1n} - q_{2i}^*)} = s_o^*. \tag{4.17}$$

(b) We have

$$q_{2i}^* \begin{cases} > q_{2o}^*, & q_{1n} > \tilde{q}_{1n}; \\ = q_{2o}^*, & q_{1n} = \tilde{q}_{1n}; \\ < q_{2o}^*, & q_{1n} < \tilde{q}_{1n}, \end{cases} \tag{4.18}$$

where $\tilde{q}_{1n}$ is uniquely determined by

$$\frac{q_{2i}^*(\alpha_o s_o^* + \gamma)}{\alpha_i(\beta\tilde{q}_{1n} - q_{2i}^*)} = \frac{\alpha_o s_o^*}{\alpha_i}. \tag{4.19}$$

(c) If $\alpha_o \geq \alpha_i$, then $\hat{q}_{1n} \geq \tilde{q}_{1n}$. Otherwise, $\hat{q}_{1n} < \tilde{q}_{1n}$.

Proof. See Appendix 4.B. ■

As Theorem 4.4.4 shows, the acquisition price offered by the OEM can be both larger and smaller than that of the IO, and the same holds for its returns market share. There are threshold values for the production of new products in period 1, above which the OEM offers a higher acquisition price acquires more cores than the IO in period 2.

In Section 4.5, we will derive further sensitivity results in a numerical study. Next, we derive the optimal decision for the first period (given the optimal policies in period 2).

4.4.2 Optimal production in period 1

Using Theorem 4.4.1, the total profit for the OEM can be written as

$$\Pi_o(q_{1n}) = \delta_1(M_1 - q_{1n})q_{1n} - c_n q_{1n} + \Pi_{2o}^*(q_{1n}), \tag{4.20}$$

where

$$\Pi_{2o}^*(q_{1n}) = \frac{\delta_2}{4}\left((M_2 - q_{2i}^*)^2 - \left(\frac{c_n}{\delta_2}\right)^2\right) - c_n q_{2n}^* - \frac{c_n + c_o}{2} q_{2o}^*. \tag{4.21}$$

The following theorem states that Π_o is concave and provides the first order condition for finding the optimal value of q_{1n}.

Theorem 4.4.5 *Under the condition*

$$4\frac{M_2}{M_1} > \beta^2 + \frac{6\beta^2 abM_2(1 - c_o)}{(a + b - 3\beta M_1)^3} \tag{4.22}$$

	M_1	M_2	β	α_o	α_i	c_n	c_o	c_i
Low	10	10	0.1	0.01	0.01	0.7	0.01	0.01
Medium	50	50	0.3	1	1	0.8	0.35	0.35
High	200	200	0.5	100	100	0.9	0.65	0.65

Table 4.2: Parameter values considered in the numerical investigation.

the total profit Π_o *for the OEM is strictly concave in* q_{1n}*, and the first order condition is given by*

$$\delta_1(M_1 - 2q_{1n}) - c_n - \frac{\delta_2}{2} q_{2i}^{*\prime}(M_2 - q_{2i}^*) + \frac{c_o \alpha_o s_o^* + c_n \gamma}{2(\alpha_o s_o^* + \gamma)} q_{2i}^{*\prime} + \beta \frac{c_n - c_o}{2} \frac{\alpha_o s_o^* \gamma}{\alpha_o s_o^* + \gamma} = 0. \tag{4.23}$$

Proof. See Appendix 4.B. ■

Due to the complexity of the optimality condition for q_{1n} in Theorem 4.4.5, this does not provide direct insights. However, we can analytically derive the effect of M_1 on the optimal production quantities and acquisition prices. It is found that an increase in M_1 leads to more manufacturing in period 1, and to more remanufacturing and less manufacturing in period 2. Also, higher manufacturing implies more available cores and so, the acquisition price of the IO decreases. (See Appendix 4.C.) In Section 4.5, we will derive further insights from a numerical study. In that numerical study, we compare the profit of the OEM under two scenarios: with remanufacturing and without remanufacturing. The comparison allows us to answer the question how remanufacturing in a competitive market affects the bottom line of an OEM.

4.5 Numerical study

As stated before, the optimality condition for q_{1n} given in Theorem 4.4.5 is too complex to obtain direct insights. In this section, our objective is to obtain insights regarding both the individual and combined effects of parameters on the production decisions and profitability of remanufacturing. To achieve that, we consider a full factorial design with three different values for 8 parameters (M_1, M_2, β, α_o, α_i, c_n, c_o and c_i) namely, low, medium and high while keeping the value of γ at 0.3. All parameter values are listed in Table 4.2.

In total this gives 6561 potential data scenarios, but we only retained the scenarios for which manufacturing and remanufacturing both take place. Based on the filtered data, we depicted

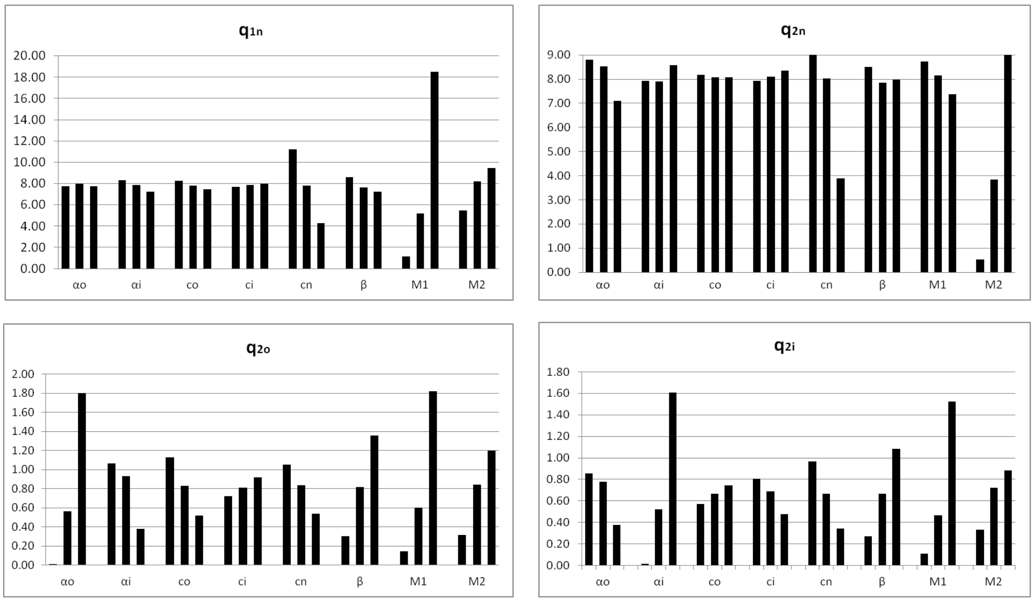


Figure 4.1: Impact of parameters on average production quantities (Parameter values are ordered as low, medium and high).

the average quantities for each parameter when it is low, medium and high values of parameters (from left to right) in Figure 4.1.

From Figure 4.1, we find that larger market M_1 in period 1 leads to more manufacturing q_{1n} in that period. This implies that more cores are available in period 2, which leads to more remanufacturing by both the OEM and IO, and less manufacturing by the OEM. An increased production cost leads to lower manufacturing levels in both periods and less remanufacturing in the second period. The reason is that costly manufacturing of new products leads to less production in the first period, which implies fewer available cores in period 2. Low β values also mean fewer returns and less remanufacturing in period 2. We see that there is a symmetry in the effects of the parameters on q_{2o} and q_{2i}. For instance, higher remanufacturing cost for the OEM or lower remanufacturing cost for the IO implies that the OEM is less competitive on remanufacturing, which results in less remanufacturing by the OEM and more by the IO. In

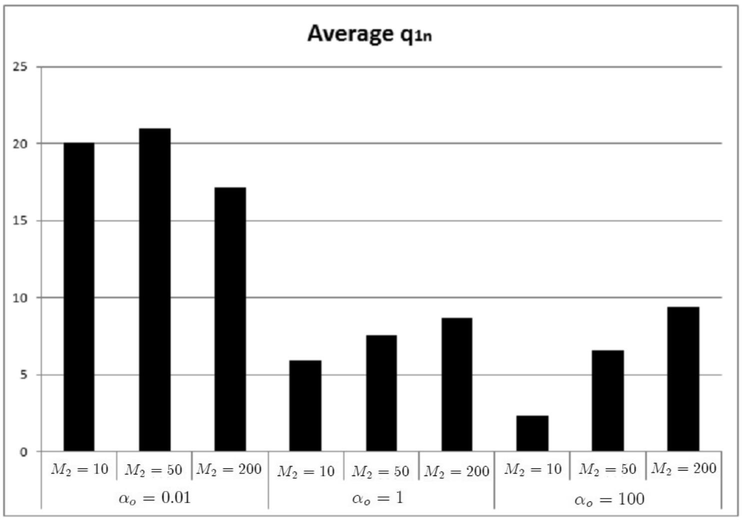


Figure 4.2: Changes in average q_{1n} with respect to α_o and M_2 when $\alpha_i = 100$.

the reverse case, i.e., when remanufacturing is more costly for the IO and/or less costly for the OEM, the IO remanufactures less and OEM remanufactures more. In Figure 4.1 we also see that the collection coefficients α_o and α_i have a significant effect on the remanufacturing quantities. An increase in α_o or a decrease in α_i makes it easier for the OEM to acquire cores relative to the IO, and therefore leads to more remanufacturing and less manufacturing by the OEM in the second period and less remanufacturing by the IO. On the contrary, the IO remanufactures more if α_o is low and/or α_i is high. Also when α_i is high, the OEM produces less in the first period to provide fewer cores to the IO.

From Figure 4.1, we see that a larger market size in period 2 leads to more production in period 1. The reason is that more manufacturing in the first period increases the availability of cores in the second period, and thereby the potential cost savings from remanufacturing. Even though q_{1n} increases with M_2 on average, there are some instances where we observe the opposite. In Figure 4.2, we analyze the change in the average production amount in the first period with respect to M_2 and α_o when the IO has a high collection coefficient ($\alpha_i = 100$). We find that when it is difficult for the OEM to collect the available cores ($\alpha_o = 0.01$), an increase in the market size in the second period leads lower manufacturing level q_{1n} to reduce the market

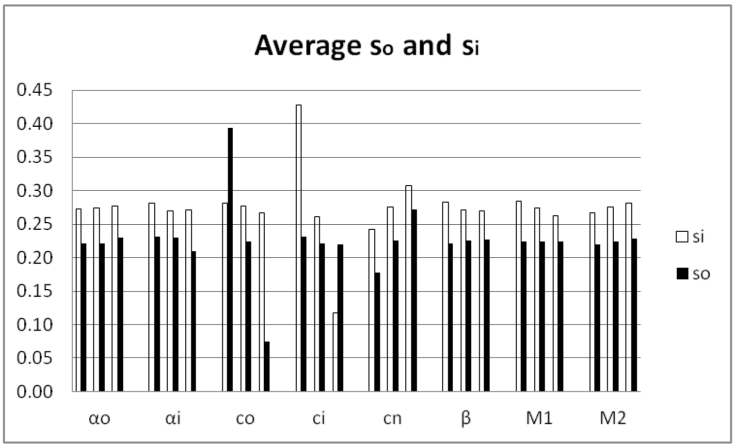


Figure 4.3: Impact of parameters on s_o and s_i (Parameter values are ordered as low, medium and high).

share of the IO in the second period.

The effect of parameters on the acquisition prices is presented in Figure 4.3. Note from this figure that, as stated in Theorem 4.4.4, the acquisition price of the OEM can be both larger and smaller than that of the IO. As expected, the acquisition price for the IO decreases if his cost of remanufacturing increases. It also decreases if the cost of remanufacturing for the OEM increases, as that leads to a lower acquisition price from the OEM and hence less competition for cores. An increase in c_n makes remanufacturing more profitable for the OEM, which therefore increases its acquisition price (as also follows from Theorem 4.4.1). The increased competition forces the IO to also raise its acquisition price. An increase in the market size in period 2 implies that a higher sales price can be achieved, to which the IO reacts by raising its acquisition price so that it obtains more cores. This also provides further explanation for the, earlier discussed, counter-intuitive result that the OEM produces less new items in the first period if M_2 increases. The IO raises the acquisition price if M_2 increases, thereby increasing its market share, and hence the OEM lowers q_{1n} to protect its market share.

Remanufacturing allows the OEM to reduce its production cost. However, it also allows the IO to take over part of the market. So, what is the overall effect of having a remanufacturing

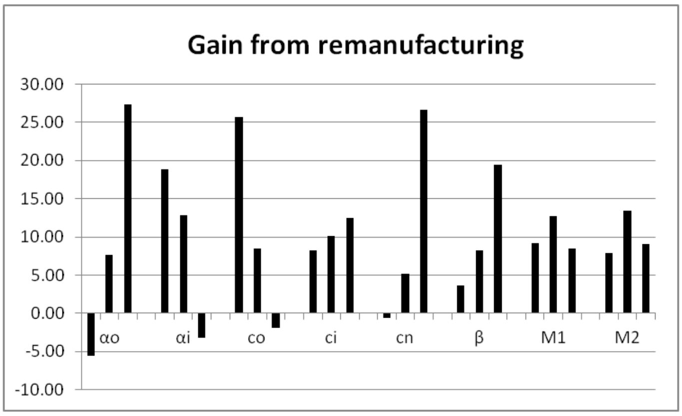


Figure 4.4: Effects of parameters on the average gain (Parameter values are ordered as low, medium and high).

option on the OEM's profit? This question is especially relevant if the OEM can prevent remanufacturing by for instance not designing for remanufacturing or making frequent changes to a product as done by Lexmark, a well-known toner cartridges manufacturer. Lexmark offers pre-bates to customers that ensures to return used cartridges. More recently, Lexmark changed the design of its product by installing a chip to avoid remanufacturing by third parties.

We address the above question by comparing the OEM's profit with remanufacturing as determined in previous sections to the maximum profit without remanufacturing that is determined next. For the non-remanufacturing model, the OEM's profit is given by

$$\delta_1(M_1 - q_{1n})q_{1n} + \delta_2(M_2 - q_{2n})q_{2n} - c_n(q_{1n} + q_{2n}).$$

It is easy to see that the optimal manufacturing quantities are

$$\tilde{q}_{1n} = \frac{M_1(1-c_n)}{2}, \tilde{q}_{2n} = \frac{M_2(1-c_n)}{2}.$$

The corresponding optimal profit is given by

$$\tilde{\Pi}_o = \frac{(M_1 + M_2)(1-c_n)^2}{4}. \tag{4.24}$$

In Figure 4.4 we analyze the effects of parameters on the average percentage gain. The percentage gain for the OEM from remanufacturing is calculated as $100(\Pi_o^* - \tilde{\Pi}_o)/\tilde{\Pi}_o$ where $\tilde{\Pi}_o$

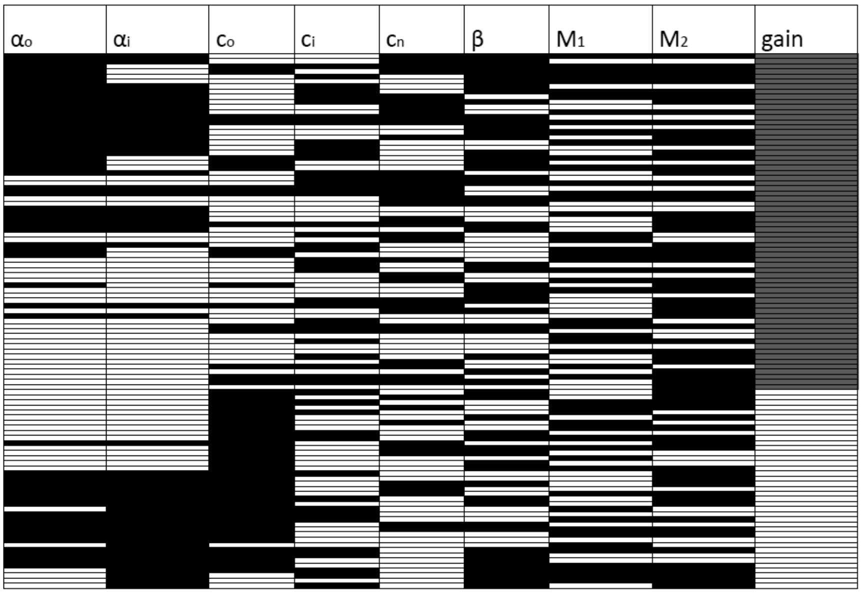


Figure 4.5: Combined effects of parameters on percentage gain. The percentage gain is sorted in decreasing order where positive values are colored gray in the "gain" column. For the parameters, black cells represent high values and white cells represent low values.

is given as in (4.24). As depicted in Figure 4.4, remanufacturing is a not a profitable option for the OEM when the cost benefit of remanufacturing is less pronounced (high remanufacturing cost or low production cost) and it is difficult for the OEM to collect available cores (low α_o or high α_i). Also, the gain first increases then decreases with market sizes M_1 and M_2. This is again due to the fact that competition is more intense in period 2 as more cores are available and the market is bigger in the second period.

After discussing the individual effects of the different model parameters, we turn to their combined effects. Figure 4.5 depicts the combined effects of parameters on gain. The percentage gain is sorted in decreasing order in Figure 4.5 where positive values are colored gray in the "gain" column. For the parameters, black cells represent high values and white cells represent low values. For readability, medium values are not included in this figure. We find that approximately the top 25% gain is reached when α_o is high and the lowest 25% gain is obtained when α_i is high indicating that these parameters pay a crucial role. Also, as expected, remanufacturing is seldom beneficial when the remanufacturing is relatively costly for the OEM. Additionally, when the OEM has an advantage on core acquisition, the gain is particularly high, if market sizes for period 1 and 2 are similar. The explanation is that in such cases, there is sufficient market in period 2 to sell a relatively (compared to total production over the life-cycle) large number of remanufactured products. If the market in period 2 is much higher than in period 1, all available cores can still be remanufactured, but relatively few cores are available and so the profit gain is smaller. The gain is generally even smaller or negative if the market size is much larger in period 1, as many returns arrive "too late" in the sense that the market is already declining. Finally, the low gains are observed when it is more likely for the IO to collect fewer available cores (high β) and the cost benefit of remanufacturing is less significant (low c_n).

4.6 Different new and remanufactured products

In this section, we relax the assumption that new and remanufactured products are indistinguishable, and instead assume that consumers have a different willingness to pay for new and remanufactured products. We mainly consider the case where the remanufactured products of the OEM and IO are sold at the same price and at the end of this section we discuss the case where consumers have higher willingness to pay for the OEMs remanufactured product.

A consumer of type θ (i.e., willingness to pay for new product equal to θ) has a willingness to pay for the remanufactured product equal to $\Psi\theta$, where $0 \leq \Psi \leq 1$. Then the inverse demand functions can be written as follows,

$$\begin{aligned} p_{2n} &= \delta_2(M_2 - q_{2n} - \Psi(q_{2o} + q_{2i})), \\ p_{2r} &= \delta_2\Psi(M_2 - q_{2n} - (q_{2o} + q_{2i})). \end{aligned}$$

The derivation for these functions is provided in Appendix 4.A. The OEM's profit in period 2 is

$$\Pi_{2o} = p_{2n}q_{2n} + p_{2r}q_{2o} - c_n q_{2n} - c_o q_{2o} - s_o r_o. \tag{4.25}$$

The IO only operates in period 2 and its profit is given by

$$\Pi_i = \Pi_{2i} = p_{2r}q_{2i} - c_i q_{2i} - s_i r_i. \tag{4.26}$$

As in the case with indistinguishable new and remanufactured product; neither player has an incentive to acquire returns that it will not remanufacture. Therefore, $q_{2o} = r_o$ and $q_{2i} = r_i$ still hold. Additionally, similar to the previous case, the profit function (4.26) is concave in q_{2i}, and (4.25) is jointly concave in (q_{2n}, q_{2o}). (See Appendix 4.B.) Hence, there exists a unique Nash Equilibrium $(q^*_{2n}, q^*_{2o}, q^*_{2i})$ given by the following first-order conditions:

$$\delta_2(M_2 - 2q^*_{2n} - 2\Psi q^*_{2o} - \Psi q^*_{2i}) - c_n = 0, \tag{4.27}$$

$$\delta_2\Psi(M_2 - 2q^*_{2n} - 2q^*_{2o} - q^*_{2i}) - c_o - 2s^*_o = 0, \tag{4.28}$$

$$\delta_2\Psi(M_2 - q^*_{2n} - q^*_{2o} - 2q^*_{2i}) - c_i - 2s^*_i = 0. \tag{4.29}$$

Using (4.27) and (4.28), we find the optimal acquisition price offered by the OEM as follows

$$s^*_o = \frac{c_n - c_o}{2} - \frac{(1-\Psi)\delta_2(M_2 - 2q^*_{2n})}{2}.$$

Note that in this case, s^*_o depends not only on the cost structure but also on the market size and the optimal manufacturing amount in the second period. Since the manufacturing amount affects the prices of new as well as remanufactured products, the marginal revenue from remanufacturing is different from the case with identical new and remanufactured products discussed in the previous sections. This explains why s^*_o no longer depends on c_n and c_o only. Finding the analytical optimal solution for the second period is complex in this case, and thus we conduct a numerical study. Subramanian and Subramanyam (2012) compare online prices of new and remanufactured products, and find that Ψ ranges from 0.60 to 0.85. Thus, in the numerical study we take $\Psi = 0.75$ and for other parameters we use the values shown in Table 4.2. Figure 4.6 depicts the average values for the decision variables when new and remanufactured products sold at the same price (represented by non-filled bars on left) or sold at different prices (represented by filled bars on right).

From Figure 4.6, we observe similar trends for both situations concerning the effects of parameters on the production quantities. For the acquisition prices, we find that both acquisition prices are lower when the remanufactured products are sold at lower prices. This is due to the fact that the marginal value for selling a remanufactured product will be lower when the remanufactured products are indistinguishable.

In Figure 4.7, we analyze the change in the average production amount in the first period for both cases with respect to M_2 and α_o when the IO has a high collection coefficient ($\alpha_i = 100$), as we did in Section 6. Also for the case of differently perceived new and remanufactured products, we find that when it is difficult for the OEM to collect the available cores ($\alpha_o = 0.01$), an increase

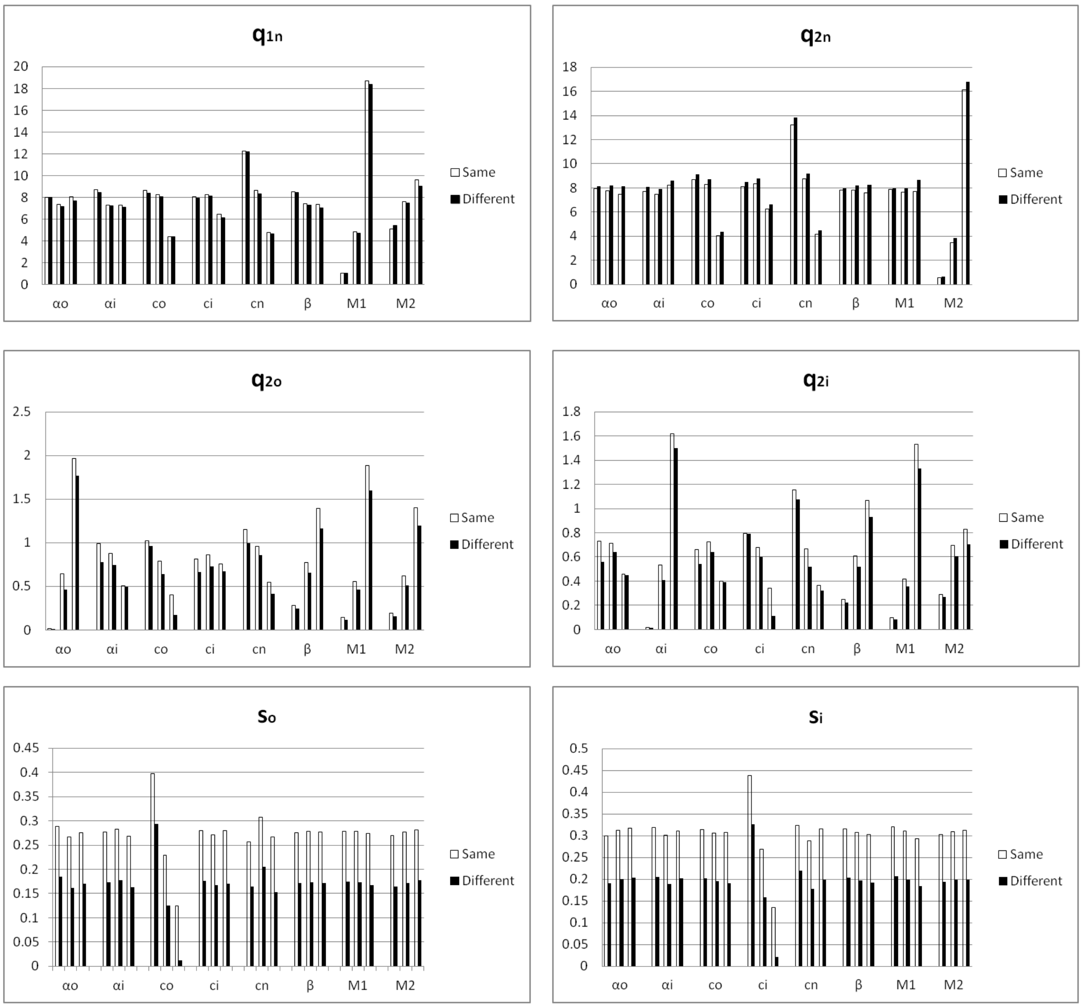


Figure 4.6: The changes in the production quantities and acquisition prices when new and remanufactured products are sold at the same or different prices.

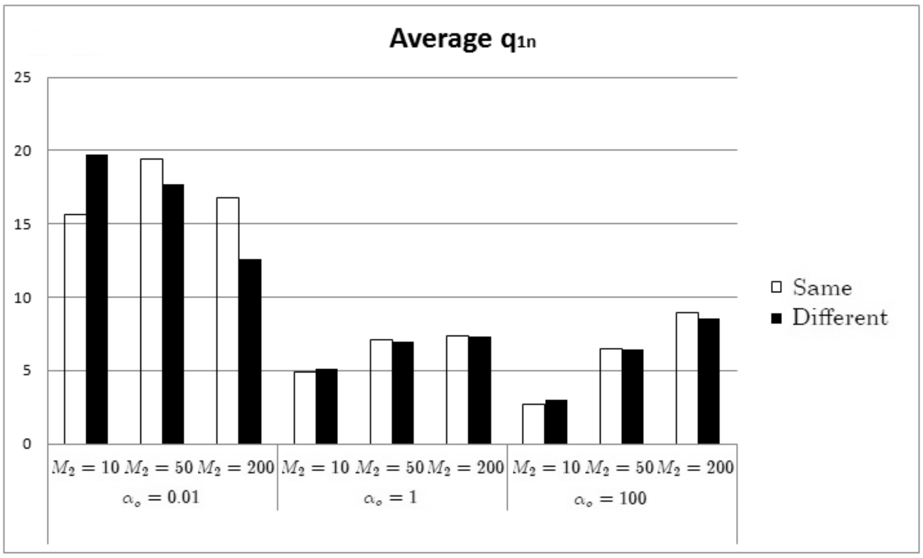


Figure 4.7: Changes in average q_{1n} with respect to α_o and M_2 when $\alpha_i = 100$.

in the market size in the second period leads a lower manufacturing level, q_{1n}, in period 1 in order to reduce the market share of the IO in the second period.

Figure 4.8 compares the average percentage gain when new and remanufactured products are sold at the same price (represented by non- filled bars on left) or sold at different prices (represented by filled bars on right). We find that in general the percentage gain is smaller when the remanufactured products are distinguishable. Also, the change in the percentage gain with different values for each parameter is similar for distinguishable and indistinguishable remanufactured product cases.

Figure 4.9 depicts a special case in Figure 4.8 where $\alpha_i = 100$, i.e., it is easy for IO to collect the available cores. We find that for the situations where remanufacturing is not profitable (compared to manufacturing only), the profit loss decreases when the products are sold at different prices.

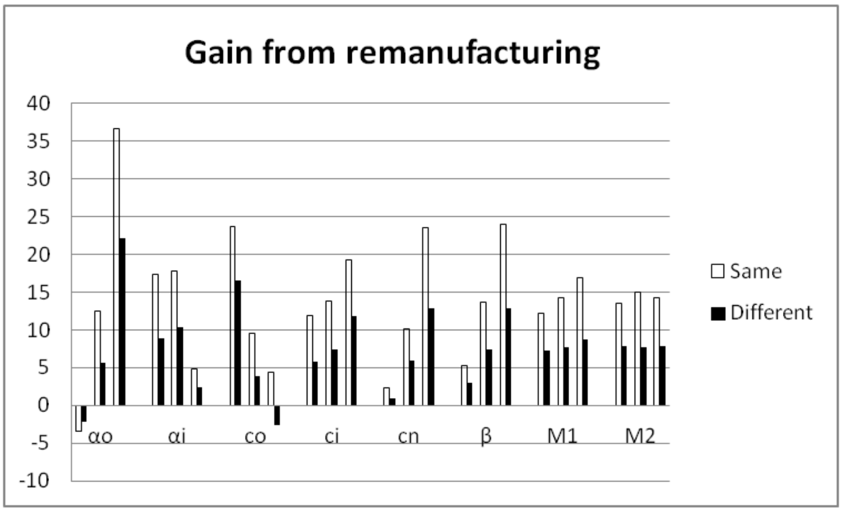


Figure 4.8: The changes in the average gain when new and remanufactured products are sold at the same or different prices.

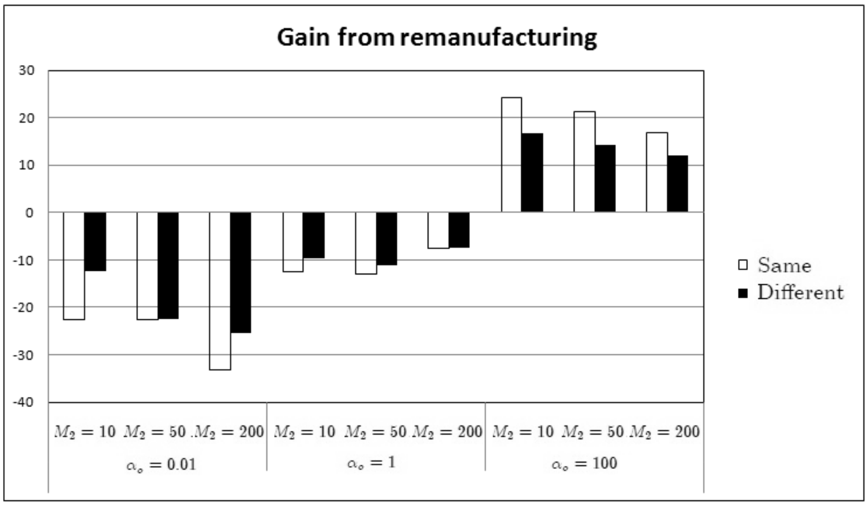


Figure 4.9: The changes in the average gain when new and remanufactured products are sold at the same or different prices and $\alpha_i = 100$.

4.6.1 OEM's remanufactured products are more valuable than IO's

So far, in this section, we assumed that consumers have a different willingness to pay for new and remanufactured products where consumers have the same willingness to pay for the OEM's and IO's remanufactured product. However, it is also possible that a consumer has a higher willingness to pay for the OEM's product. To obtain insights for this situation, we assume that the new and remanufactured products of the OEM are sold at the same price, p_{2n}, and the remanufactured products of the IO is sold at a lower price, p_{2r}, and we conduct a numerical study using the same parameter values shown in Table 4.2 and take $\Psi = 0.75$. The details of the analysis and the results of the numerical study are provided in Appendix 3.D. We observe that also for this case, the optimal acquisition price for the OEM only depends on its cost structure. We compare the optimal production quantities and acquisition prices in this case with the main case where new and remanufactured products are sold at the same price. We find that the effects of the parameters on the optimal production quantities and acquisition prices are similar. The average gain in this case is more than the case that the OEM's remanufactured products are also sold a discounted price. This implies that the effects of the parameters on the optimal production quantities and profitability of remanufacturing will be similar when the remanufactured products of the OEM and IO are sold at different discounted prices.

4.7 Conclusions

In this chapter we analyze the effect of competition with an IO on the remanufacturing strategy of an OEM in a two-period setting. Different from the current literature we consider competition between the parties for collecting cores, next to the competition for market share. We determine the existence of a unique Nash equilibrium between the parties in period 2 and find the optimal acquisition prices and (re)manufacturing quantities. Then, we consider period 1 and determine the optimal manufacturing quantity for the OEM.

In the main part of the chapter, we analyze the case in which the new and remanufactured products are sold for the same price. Analytical and numerical sensitivity analyses lead to several managerial insights for this case. First of all, we find that OEM's acquisition price only depends on its cost structure, not on IO's acquisition price. In this sense, the OEM does not compete for collecting cores. From the numerical analysis, we see that remanufacturing is a profitable option for the OEM when it has a dominance in collecting the available cores and remanufacturing leads cost savings. We also see that, when the cost benefits of remanufacturing diminishes and the IO has more chance to collect the available cores, the OEM manufactures less in the first period as the market in the second period gets larger to protect its overall market share.

We then consider the case where consumers have lower willingness to pay for the remanufactured products. We find that the effect of the parameters on the optimal production quantities and acquisition prices are similar with the case of indistinguishable remanufactured products. We also find that the acquisition prices offered both by the OEM and IO decreases, and remanufacturing becomes less profitable overall.

In this chapter we make some limiting assumptions. First of all, we consider a two-period problem where the OEM and the IO competes both to acquire available cores and to sell products in the second period. For multiple period cases, the problem would be intractable since the concavity of the profit functions is lost or at least cannot easily be proven. We also assumed that the consumers' willingness to pay for the products is uniformly distributed which leads to linear inverse demand functions. In further research, other distributions can also be used for modeling consumers' willingness to pay for the products. Other possible research areas are considering random demand and competition between more than two parties can be considered. Finally, capacity restrictions can be included.

Appendix 4.A. Derivation of inverse demand functions

Net utility of a consumer, NU, is

$$NU = \Psi^m \theta - p_{2z},$$

where $m = 0$ and $z = n$ if the product is new, and $m = 1$ and $z = r$ if the product is remanufactured. A consumer has 3 different strategies, based on the value of NU: purchase a new product, strategy N; purchase a remanufactured product, strategy R; and purchase nothing, strategy X. First, consider the lowest valuation consumer who adopts an R strategy i.e., the consumer who is indifferent between strategy R and X. In this case, $\theta = \delta_2(M_2 - q_{2n} - q_{2r})$

where $q_{2r} = q_{2o} + q_{2i}$.

$$NU_R = \delta_2 \Psi(M_2 - q_{2n} - q_{2r}) - p_{2r} = 0 = NU_X$$

where the net utility of the consumer from strategy R is NU_R, and the net utility of the consumer from strategy X is NU_X. This implies $p_{2r} = \delta_2 \Psi(M_2 - q_{2n} - q_{2r})$. Then, consider the lowest valuation consumer who adopts an N strategy i.e., the consumer who is indifferent between strategy N and R. In this case, $\theta = \delta_2(M_2 - q_{2n})$.

$$NU_N = \delta_2(M_2 - q_{2n}) - p_{2n} = \delta_2 \Psi(M_2 - q_{2n}) - p_{2r} = NU_R$$

where NU_N is the net utility of the consumer from strategy N $p_{2r} = \delta_2 \Psi(M_2 - q_{2n} - q_{2r})$. This implies $p_{2n} = \delta_2(M_2 - q_{2n} - \Psi q_{2r})$.

Appendix 4.B. Proof of theorems

Proof of joint concavity of (4.5)

The second order derivatives of (4.5) with respect to q_{2n} and q_{2o} are as follows,

$$\begin{aligned} \frac{\partial^2 \Pi_{2o}}{\partial q_{2n}^2} &= -2\delta_2, \frac{\partial^2 \Pi_{2o}}{\partial q_{2o}^2} = -2\delta_2 - \frac{2\alpha_i s_i}{\alpha_o q_{2i}}, \\ \frac{\partial^2 \Pi_{2o}}{\partial q_{2n} \partial q_{2o}} &= \frac{\partial^2 \Pi_{2o}}{\partial q_{2o} \partial q_{2n}} = -2\delta_2. \end{aligned}$$

From, these derivatives we construct the Hessian for (4.5) as

$$H = \begin{vmatrix} -2\delta_2 & -2\delta_2 \\ -2\delta_2 & -2\delta_2 - \frac{2\alpha_i s_i}{\alpha_o q_{2i}} \end{vmatrix}.$$

It can easily be seen that the Hessian is negative definite since $-2\delta_2 < 0$ and $4\delta_2 \alpha_i s_i / \alpha_o q_{2i} > 0$. Thus, (4.5) is jointly concave with respect to q_{2n} and q_{2o}.

Proof of Theorem 4.4.1

Combining (4.8) and (4.9) gives (4.11). By substituting (4.8) into (4.10), we have

$$M_2 - 3q_{2i}^* + (c - 2c_i)/\delta = 4s_i/\delta. \tag{4.30}$$

By substituting (4.4) into (4.30), we have

$$q_{2i}^* = \frac{a + b + 3\beta q_{1n} \pm \sqrt{(a + b + 3\beta q_{1n})^2 - 12a\beta q_{1n}}}{6},$$

where $a = M_2 + (c - 2c_i)/\delta$ and $b = 4(\alpha_o s_o^* + \gamma)/(\alpha_i \delta)$.

Note that by (4.30), since $s_i \geq 0$, we require that $q_{2i} \leq a/3$. Since $q_{2i} \leq a/3$, the unique solution is given by

$$\begin{aligned} q_{2i}^* &= \frac{a + b + 3\beta q_{1n} - \sqrt{(a + b + 3\beta q_{1n})^2 - 12a\beta q_{1n}}}{6} \\ &= \frac{a + b + 3\beta q_{1n} - \sqrt{(a + b - 3\beta q_{1n})^2 + 12b\beta q_{1n}}}{6}, \end{aligned}$$

which gives (12). Based on the first-order conditions, we have

$$\begin{aligned} q_{2o}^* &= \beta q_{1n} \frac{\alpha_o s_o^*}{\alpha_o s_o^* + \alpha_i s_i^* + \gamma} \\ &= \beta q_{1n} \frac{\alpha_o s_o^*}{\alpha_o s_o^* + \gamma} \frac{\beta q_{1n} - q_{2i}^*}{\beta q_{1n}} \\ q_{2n}^* &= \frac{M_2 - 2q_{2o}^* - q_{2i}^* - c/\delta}{2} \\ s_i^* &= \frac{q_{2i}^*(\alpha_o s_o^* + \gamma)}{\alpha_i(\beta q_{1n} - q_{2i}^*)}, \end{aligned} \tag{4.31}$$

which give (4.13), (4.14) and (4.15). This completes the proof.

Proof of theorem 4.4.2

(i) We have

$$\begin{aligned} q_{2i}^{*\prime} &= 3\beta - 1/2\Delta^{-1/2}\frac{\partial \Delta}{\partial q_{1n}}, \\ q_{2i}^{*\prime} &= 3\beta - 1/2\Delta^{-1/2}(-6\beta(a - b - 3\beta q_{1n})). \end{aligned}$$

We consider two cases. Case I: $a - b - 3\beta q_{1n} \geq 0$. Then, it is clear that $q_{2i}^{*\prime} > 0$. Condition $q_{2i}^{*\prime} < \beta$ is equivalent to $\Delta^{1/2} > a - b - 3\beta q_{1n}$, which obviously holds from the definition of Δ.

Case II: $a - b - 3\beta q_{1n} < 0$. It is clear that $q_{2i}^{*\prime} < \beta$. Condition $q_{2i}^{*\prime} > 0$ is equivalent to

$$\Delta^{1/2} > b + 3\beta q_{1n} - a.$$

This can be rewritten as

$$(a + b - 3\beta q_{1n})^2 + 12b\beta q_{1n} \quad > \quad (b + 3\beta q_{1n} - a)^2,$$

or $4ab > 0$, which obviously holds. In summary, for both cases, $0 < q_{2i}^{*\prime} < \beta$. Thus, q_{2i}^* is increasing in q_{1n}.

(ii) This follows from (4.14), (4.15), and part (i) of this theorem.

(iii) From (4.8) we have

$$q_{2n}^* + q_{2o}^* = \frac{M_2 - c_n/\delta_2 - q_{2i}^*}{2}.$$

Combining this with (i) of this theorem, it is clear that $(q_{2n}^* + q_{2o}^*)$ is decreasing in q_{1n}.

(iv) From (4.8) we have

$$q_{2n}^* + q_{2o}^* + q_{2i}^* = \frac{M_2 - c_n/\delta_2 + q_{2i}^*}{2}.$$

which is increasing in q_{1n}.

(v) This follows directly from Theorem 4.4.1.

(vi) From (13) we get

$$s_i^{*\prime} = \beta \frac{\alpha_o s_o^* + \gamma}{\alpha_i(\beta q_{1n} - q_{2i}^*)^2}(q_{1n} q_{2i}^{*\prime} - q_{2i}^*).$$

Therefore, showing that s_i^* is decreasing in q_{1n} is equivalent to showing that $q_{1n} q_{2i}^{*\prime} - q_{2i}^* < 0$.

We have

$$\begin{aligned}
& q_{1n} q_{2i}^{*\prime} - q_{2i}^* \\
= \; & \beta q_{1n} \frac{\Delta^{1/2} + [(a+b-3\beta q_{1n}) - 2b]}{2\Delta^{1/2}} - \frac{a+b+3\beta q_{1n} - \sqrt{(a+b-3\beta q_{1n})^2 + 12b\beta q_{1n}}}{6} \\
= \; & \frac{(a+b)[(a+b-3\beta q_{1n}) - \Delta^{1/2}] + 6\beta b q_{1n}}{6\Delta^{1/2}}.
\end{aligned}$$

By simple algebra, we can show that

$$(a+b)[(a+b-3\beta q_{1n}) - \Delta^{1/2}] + 6\beta b q_{1n} < 0.$$

Thus, we have $q_{1n} q_{2i}^{*\prime} - q_{2i}^* < 0$, and hence s_i^* is decreasing in q_{1n}.

Proof of theorem 4.4.3

First, we show that $q_{2i}^*/(\beta q_{1n})$ is decreasing in q_{1n}. By (4.12), we have

$$\begin{aligned}
(q_{2i}^*/\beta q_{1n})' &= \frac{q_{2i}^{*\prime} q_{1n} - q_{2i}^*}{\beta q_{1n}^2} \\
(q_{2i}^*/q_{1n})' \beta q_{1n}^2 &= \beta q_{1n} \frac{\Delta^{1/2} + (a-b-3\beta q_{1n})}{2\Delta^{1/2}} - \frac{a+b+3\beta q_{1n} - \Delta^{1/2}}{6} \\
&\sim \Delta^2 + 3\beta q_{1n}(a-b-3\beta q_{1n}) - (a+b)\Delta^{1/2}.
\end{aligned}$$

By simple algebra, it is not difficult to show that $\Delta^2 + 3\beta q_{1n}(a-b-3\beta q_{1n}) - (a+b)\Delta^{1/2} < 0$. Thus, $(q_{2i}^*/\beta q_{1n})' < 0$. By (14), it is clear that $q_{2o}^*/(\beta q_{1n})$ is increasing in q_{1n} since $(q_{2i}^*/\beta q_{1n})' < 0$. Finally, we have

$$q_{2o}^* + q_{2i}^* = \frac{\alpha_o r_o^*}{\alpha_o r_o^* + K}\beta q_{1n} + \frac{K}{\alpha_o r_o^* + K} q_{2i}^*,$$

which gives

$$\frac{q_{2i}^*+q_{2o}^*}{\beta q_{1n}}=\frac{\alpha_o r_o^*}{\alpha_o r_o^*+K}+\frac{K}{\alpha_o r_o^*+K}\frac{q_{2i}^*}{\beta q_{1n}}.$$

Since, $(q_{2i}^*/\beta q_{1n})'<0$, as shown above $(q_{2i}^*+q_{2o}^*)/(\beta q_{1n})$ is also decreasing in q_{1n}.

Proof of theorem 4.4.4

Part (a) follows from (4.11), (4.13) and Theorem 4.4.3 (vi). Part (b) follows from similar arguments and by using (4.7). Combining parts (a) and (b) leads to part (c).

Proof of theorem 4.4.5

The first-order condition (4.23) follows directly from (4.20) and (4.21). By taking the second derivative of Π_o with respect to q_{1n}, we have

$$\frac{d^2\Pi_o}{dq_{1n}^2}=-\frac{\delta_2}{2}\left\{4\frac{\delta_1}{\delta_2}-\left(\frac{\partial q_{2i}^*}{\partial q_{1n}}\right)^2+\frac{\partial^2 q_{2i}^*}{\partial q_{1n}^2}\left[M_2-q_{2i}^*-\frac{c_o\alpha_o s_o^*+c_n\gamma}{\delta_2(\alpha_o s_o^*+\gamma)}\right]\right\}. \tag{4.32}$$

where $\delta_1=1/M_1$ and $\delta_2=1/M_2$. We also have

$$\frac{\partial q_{2i}^*}{\partial q_{1n}} = \beta\frac{\Delta^{1/2}+(a-b-3\beta q_{1n})}{2\Delta^{1/2}}, \tag{4.33}$$

$$\frac{\partial^2 q_{2i}^*}{\partial q_{1n}^2} = -\beta^2\frac{6ab}{\Delta^{3/2}}, \tag{4.34}$$

where $\Delta=(a+b-3\beta q_{1n})^2+12b\beta q_{1n}$. It can easily be seen that $d^2\Pi_o/dq_{1n}^2<0$ holds when

$$4\frac{M_2}{M_1}-\left(\frac{\partial q_{2i}^*}{\partial q_{1n}}\right)^2+\frac{\partial^2 q_{2i}^*}{\partial q_{1n}^2}\left[M_2-q_{2i}^*-\frac{c_o\alpha_o s_o^*+c_n\gamma}{\delta_2(\alpha_o s_o^*+\gamma)}\right]>0. \tag{4.35}$$

In Theorem 2, we proved that $0<\partial q_{2i}^*/\partial q_{1n}<\beta$ holds. Also $\partial^2 q_{2i}^*/\partial q_{1n}^2<0$. Let us focus on $M_2-q_{2i}^*-(c_o\alpha_o s_o^*+c_n\gamma/\delta_2(\alpha_o s_o^*+\gamma))$. We have

$$M_2-q_{2i}^*-\frac{c_o\alpha_o s_o^*+c_n\gamma}{\delta_2(\alpha_o s_o^*+\gamma)}<M_2(1-c_o)-q_{2i}<M_2(1-c_o) \tag{4.36}$$

The inequalities in (4.36) holds since $c_n>c_o$ and $q_{2i}\geq 0$. Then,

$$4\frac{M_2}{M_1}-\left(\frac{\partial q_{2i}^*}{\partial q_{1n}}\right)^2+\frac{\partial^2 q_{2i}^*}{\partial q_{1n}^2}\left(M_2-q_{2i}^*-\frac{c_o\alpha_o s_o^*+c_n\gamma}{\delta_2(\alpha_o s_o^*+\gamma)}\right)>4\frac{M_2}{M_1}-\beta^2-\beta^2\frac{6abM_2(1-c_o)}{\Delta^{3/2}}$$

holds. Let us define $\Delta'=\Delta-12b\beta q_{1n}=(a+b-3\beta q_{1n})^2$. We have $\Delta'<\Delta$ since $12b\beta q_{1n}>0$. Thus,

$$\begin{aligned}4\frac{M_2}{M_1}-\beta^2-\beta^2\frac{6abM_2(1-c_o)}{\Delta^{3/2}} &> 4\frac{M_2}{M_1}-\beta^2-\beta^2\frac{6abM_2(1-c_o)}{(\Delta')^{3/2}}\\ &= 4\frac{M_2}{M_1}-\beta^2-\beta^2\frac{6abM_2(1-c_o)}{(a+b-3\beta q_{1n})^3}.\end{aligned}$$

Finally, $(a+b-3\beta q_{1n}) > (a+b-3\beta M_1)$ holds, since $q_{1n} \leq M_1$ is reduced to have nonnegative selling price in period 1. Thus, (4.35) is true when $4\frac{M_2}{M_1} - \beta^2 - \beta^2 \frac{6abM_2(1-c_o)}{(a+b-3\beta M_1)^3}$ holds. This completes the proof.

Proof of joint concavity of (4.25)

The second order derivatives of (4.25) with respect to q_{2n} and q_{2o} are as follows,

$$\begin{aligned}
\frac{\partial^2 \Pi_{2o}}{\partial q_{2n}^2} &= -2\delta_2, \frac{\partial^2 \Pi_{2o}}{\partial q_{2o}^2} = -2\delta_2 \Psi - \frac{2\alpha_i s_i}{\alpha_o q_{2i}}, \\
\frac{\partial^2 \Pi_{2o}}{\partial q_{2n} \partial q_{2o}} &= \frac{\partial^2 \Pi_{2o}}{\partial q_{2o} \partial q_{2n}} = -2\delta_2 \Psi.
\end{aligned}$$

From, these derivatives we construct the Hessian for (4.25) as

$$H = \begin{vmatrix} -2\delta_2 & -2\delta_2 \Psi \\ -2\delta_2 \Psi & -2\delta_2 \Psi - \frac{2\alpha_i s_i}{\alpha_o q_{2i}} \end{vmatrix}.$$

It can easily be seen that the Hessian is negative definite since $-2\delta_2 < 0$ and $4\delta_2(\delta_2(1-\Psi) + \alpha_i s_i/\alpha_o q_{2i}) > 0$. Thus, (4.25) is jointly concave with respect to q_{2n} and q_{2o}.

Appendix 4.C. Sensitivity analysis w.r.t M_1

It is known that q_{1n}^* is the solution of

$$\delta_1(M_1 - 2q_{1n}) - c_n - \delta_2 \frac{q_{2i}^{*'}}{2}(M_2 - q_{2i}^*) + \frac{c_o \alpha_o s_o^* + c_n \gamma}{2(\alpha_o s_o^* + \gamma)} q_{2i}^{*'} + \beta \frac{c_n - c_o}{2} \frac{\alpha_o s_o^* \gamma}{\alpha_o s_o^* + \gamma} = 0.$$

Let $f(M_1, M_2, c, c_o, c_i, \alpha_i, \alpha_o, \beta, \delta, \gamma, q_{1n}^*) = (1 - 2q_{1n}/M_1) - c_n - \delta_2 \frac{q_{2i}^{*'}}{2}(M_2 - q_{2i}^*) + \frac{c_o \alpha_o s_o^* + c_n \gamma}{2(\alpha_o s_o^* + \gamma)} q_{2i}^{*'} + \beta \frac{c_n - c_o}{2} \frac{\alpha_o s_o^* \gamma}{\alpha_o s_o^* + \gamma} = 0$. Then, according to the Implicit Function Theorem, for any parameter x, $x \in \{M_1, M_2, c_n, c_o, c_i, \alpha_i, \alpha_o, \beta, \delta, \gamma\}$ we get

$$\frac{\partial q_{1n}^*}{\partial x} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial q_{1n}^*}}$$

for the points where it is proven in Theorem 4.4.5 that $\frac{\partial f}{\partial q_{1n}^*} = \frac{d^2 \Pi_o(q_{1n})}{dq_{1n}^2} < 0$. Let $f(M_1, q_{1n}^*) = \left(1 - \frac{2q_{1n}}{M_1}\right) - c_n - \delta_2 \frac{q_{2i}^{*'}}{2}(M_2 - q_{2i}^*) + \frac{c_o \alpha_o s_o^* + c_n \gamma}{2(\alpha_o s_o^* + \gamma)} q_{2i}^{*'} + \beta \frac{c_n - c_o}{2} \frac{\alpha_o s_o^* \gamma}{\alpha_o s_o^* + \gamma} = 0.$

$$\frac{\partial q_{1n}^*}{\partial M_1} = -\frac{\frac{\partial f(M_1, q_{1n}^*)}{\partial M_1}}{\frac{\partial f(M_1, q_{1n}^*)}{\partial q_{1n}^*}}$$

$\frac{\partial f(M_1, q_{1n}^*)}{\partial M_1} = 2q_{1n}/M_1 > 0$. Thus, $\frac{\partial q_{1n}^*}{\partial M_1} > 0$ i.e. q_{1n}^* increases as M_1 increases. Then, q_{2i} and q_{2o} increase, and q_{2n} and s_i decrease with M_1 by applying Theorem 2.

Appendix 4.D. Different remanufactured products by the OEM and the IO

Here, we assume that a consumer has a higher willingness to pay for the OEM's product. To obtain insights for this situation, we assume that the new and remanufactured products of the OEM are sold at the same price, p_{2n}, and the remanufactured products of the IO is sold at price p_{2r}.

Then the inverse demand functions can be written as follows,

$$\begin{aligned} p_{2n} &= \delta_2(M_2 - q_{2n} - q_{2o} - \Psi q_{2i}), \\ p_{2r} &= \delta_2\Psi(M_2 - q_{2n} - q_{2o} - q_{2i}). \end{aligned}$$

The OEM's profit in period 2 is

$$\Pi_{2o} = p_{2n}(q_{2n} + q_{2o}) - c_n q_{2n} - c_o q_{2o} - s_o r_o. \tag{4.37}$$

The IO only operates in period 2 and its profit is given by

$$\Pi_i = \Pi_{2i} = p_{2r} q_{2i} - c_i q_{2i} - s_i r_i. \tag{4.38}$$

As in the case with indistinguishable new and remanufactured product; neither player has an incentive to acquire returns that it will not remanufacture. Therefore, $q_{2o} = r_o$ and $q_{2i} = r_i$ still hold. Additionally, similar to the indistinguishable new and remanufactured product case the profit function (4.38) is concave in q_{2i}, and (4.37) is jointly concave in (q_{2n}, q_{2o}). Hence, there exists a unique Nash Equilibrium $(q^*_{2n}, q^*_{2o}, q^*_{2i})$ given by the following first-order conditions:

$$\begin{aligned} \delta_2(M_2 - 2q^*_{2n} - 2q^*_{2o} - \Psi q^*_{2i}) - c_n &= 0, & (4.39) \\ \delta_2(M_2 - 2q^*_{2n} - 2q^*_{2o} - \Psi q^*_{2i}) - c_o - 2s^*_o &= 0, & (4.40) \\ \delta_2\Psi(M_2 - q^*_{2n} - q^*_{2o} - 2q^*_{2i}) - c_i - 2s^*_i &= 0. & (4.41) \end{aligned}$$

Using (4.39) and (4.40), we find the optimal acquisition price offered by the OEM as follows

$$s^*_o = \frac{c_n - c_o}{2}.$$

In this case, s^*_o again only depends on the cost structure of the OEM. Finding the analytical optimal solution for the second period is also complex for this case, and thus we also conduct a numerical study using the same parameter values shown in Table 5.2 and take $\Psi = 0.75$. Figure 4.10 shows that the effect of the paraders on the optimal production quantities and acquisition prices are similar to the case when all remanufactured and new products in the second period are sold at the same price. Figure 4.11 shows that as expected the profitability of remanufacturing is higher for this case since the effect of competition with the IO for the market sales is decreased.

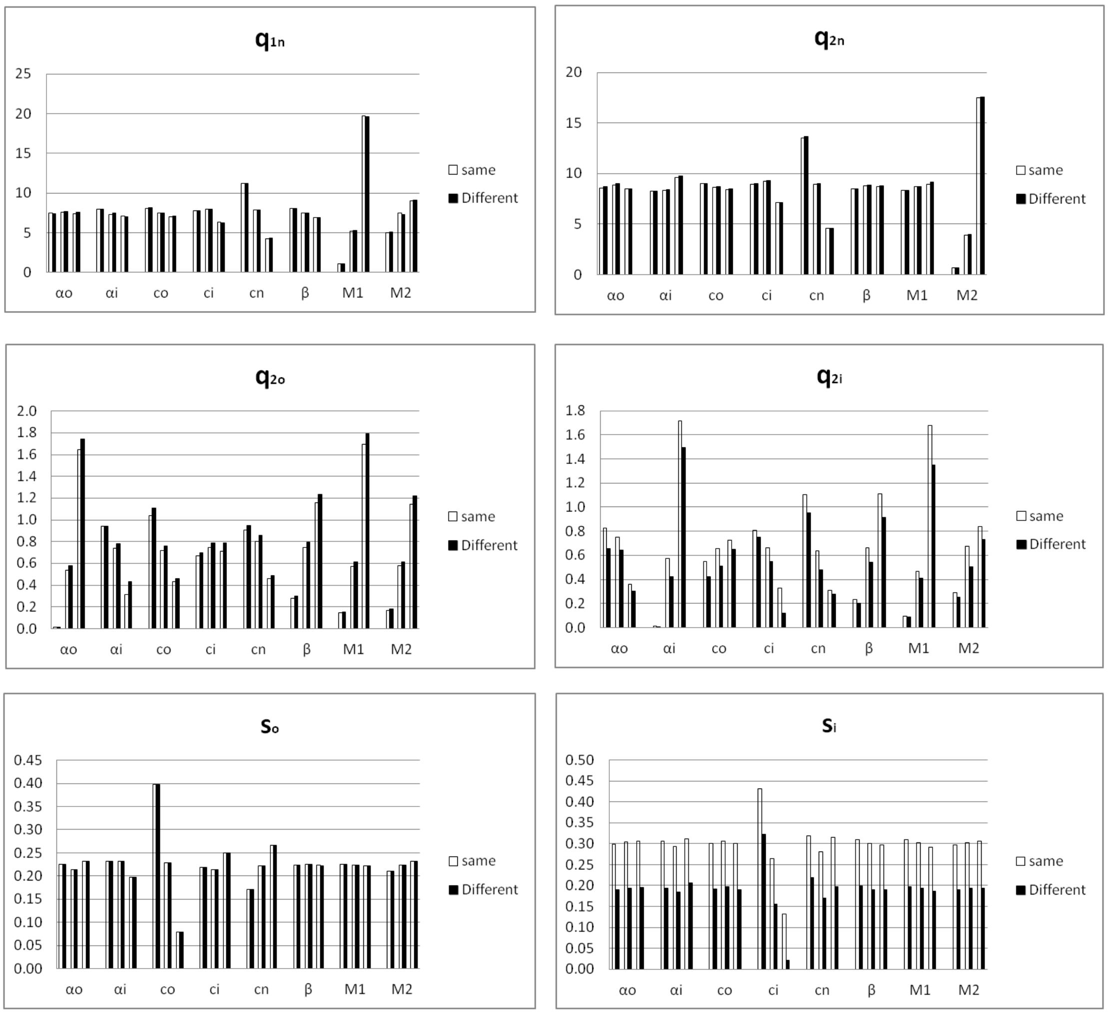


Figure 4.10: The changes in the production quantities and acquisition prices when the OEM's and IO's remanufactured products are sold at different prices.

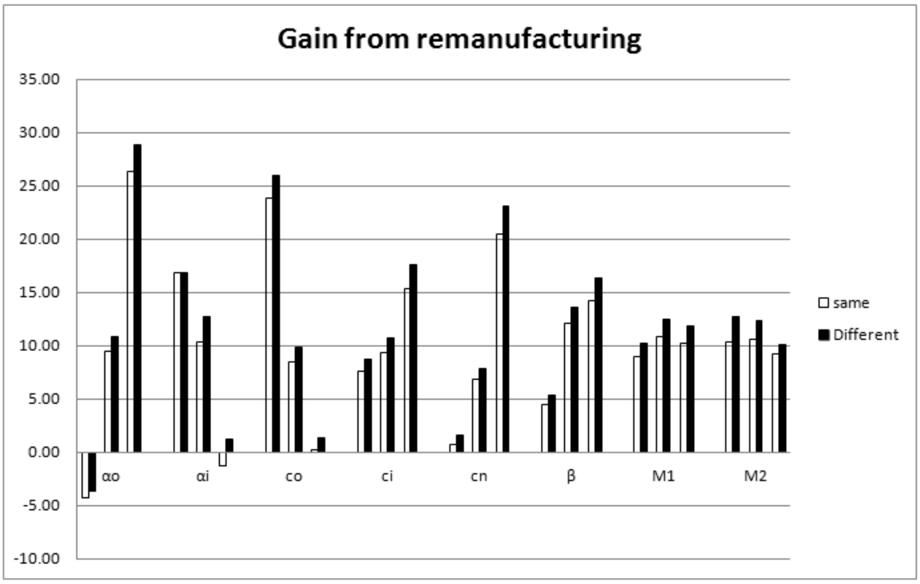


Figure 4.11: The changes in the average gain when the OEM's and IO's remanufactured products are sold at different prices.

Chapter 5

Cooperation in the returns channel

In this chapter, we analyze cooperation between original equipment manufacturers (OEMs) to increase the efficiency of the reverse channel performance by using cooperative game theory. We analyze under which conditions it is beneficial for OEMs to form a coalition in setting up a returns network, and determine stable gain allocation schemes. Important insights are that cooperation creates a significant increase in profit if OEMs have comparable revenues / savings per returned item, but not necessarily similar market shares. Moreover, cooperation is more beneficial for higher economies of scale in collection activities.

5.1 Introduction

Over the last fifty years, the level of consumption has been continuously increasing because of the rapid technological development of new products and the growing desire of consumers to acquire the latest technology. As a result, the world now faces serious environmental threats: waste and the presence of toxic materials in the discarded products (Pochampally et al., 2009). Sustainable production processes and closed loop supply chains (CLSCs) are becoming more important because of these threats. CLSC management is defined as the design, control and operation of a system to maximize value creation over the entire life cycle of the product with dynamic recovery of value from different types and volume of returns over time (Guide and Van Wassenhove, 2009). In addition to traditional forward supply chain activities, CLSCs include the following steps: collecting the used/discarded items, reverse logistics to move the used items from the points of use to points of disposition, testing, sorting, and disposition to understand the items condition and the most economically attractive reuse option, deciding on the most beneficial refurbishing option (direct reuse, repair, remanufacture, recycle or disposal), remarketing the refurbished product (Guide et al., 2003).

In this chapter we focus on collecting the used/discarded items step. Some companies collect used items directly from the customers, but others may prefer independent third parties (e.g., Genco) to handle used product collection (Kaya, 2010). Cooperation between the OEMs in collection activities is also possible, as illustrated by the following examples. In 1990, Kodak established alliances with Fuji and Konica at the collection facilities to collect returned single-used cameras. This alliance resulted in a return rate of 63% in the US, or 50 million cameras per year (Fleischmann, 2001; Aras et al., 2004). In Japan, Fuji Xerox, Ricoh and Canon have formed partnerships to collect and return each other's used photocopier machines. Six printer ink and toner ink cartridge OEMs (Epson, Canon, Hewlett-Packard, Brother, Dell, and Lexmark) have collaborated to collect used ink cartridges (Matsumoto and Umeda, 2011).

Motivated by these real life cases, in this chapter we consider cooperation between OEMs on collecting the used items, and our aim is to determine the conditions where cooperation is beneficial and to find an efficient allocation of cost savings between the parties using cooperative game theory. To the best of our knowledge, we are the first to explore cooperation between OEMs in setting up reverse logistics networks.

The rest of the chapter is organized as follows. The next section reviews the related remanufacturing and cooperative game theory literature. Section 5.3 describes the system that we analyze and provides preliminaries in the cooperative game theory. In Section 5.4, we introduce interaction between the multiple OEMs and analyze the collection game. Finally in Section 5.5, a brief summary of the findings and managerial insights are provided, and avenues for further research are discussed.

5.2 Related literature

There are numerous studies on closed-loop supply chains in the current literature. Fleischmann et al. (1997) and Guide and Van Wassenhove (2009) provide excellent reviews.

Two main streams of game theory, namely non-cooperative and cooperative game theory, have been applied widely in the supply chain management literature (Cachon and Netessine, 2004; Leng and Parlar, 2005). In CLSCs, non-cooperative games have also been addressed by several authors. Heese et al. (2005) and Atasu et al. (2008) consider competition between two OEMs. Majumder and Groeneveld (2001), Ferguson and Toktay (2006), and Ferrer and Swaminatham (2006) analyze competition between an OEM and an independent remanufacturer. Debo et al. (2005) consider competition between independent remanufacturers. Guide and Wassenhove (2006) point out that reverse supply chain coordination and incentive alignment are important research issues. Savaskan et al. (2004) and Kaya (2010) address these issues. Although they again use non-cooperative game theory, these studies are most closely linked to ours and we therefore next discuss them in more detail.

Savaskan et al. (2004) consider the problem of determining the best reverse channel structure to collect returns. They analyze three options for collecting the used-products: (1) collecting directly from the customers, (2) utilizing the retailers, (3) subcontracting the collection activity to a third party. In their scenarios they assume that new and remanufactured products are indistinguishable and that the acquisition price for the returned products is exogenous. Kaya (2010) considers coordination between a single manufacturer and a single collection agency by contracts offered by the manufacturer. In his model, the manufacturer produces both new products and remanufactured products and the collection agency manages the collection of the returns by offering incentives to customers. He determines the optimal combination of incentive and production amounts in a stochastic demand setting where the retail prices of the new and remanufactured products are exogenous.

Although cooperative game theory has many application for traditional supply chains (see also Nagarajan and Sosic (2008), Dror and Hartman (2011) and Fiestas-Janeiro (2011) for detailed reviews) and there are many cooperation cases in practice, its use is CLSCs are very limited. Gui (2012) uses notions of cooperative game theory to derive fair cost allocations for collection and recycling networks. In this chapter, we investigate whether cooperation in collecting used items leads to cost-savings for all involved parties and, if so, we try to find efficient allocations of cost savings using cooperative game theory.

5.3 System description

In this section, we describe the system and provide some preliminaries on cooperative game theory.

Let D denote the total demand for either a single OEM or multiple cooperating OEMs. We model the relationship between the investment I of one or more OEMs and the return rate τ as,

$$I = L\tau^2 D^a$$

where L is the scaling parameter to assure $0 \leq \tau \leq 1$ and a, $0 \leq a \leq 1$, is the parameter related to economies of scale. Firms that are in the same coalition will have the same return rates since they use same channels and facilities to collect the used/discarded items.

Savaskan et al. (2004) note that similar forms of response functions without economies of scale have been widely used in the advertising response models of consumer retention and product awareness, and also in the marketing literature as sales force effort response models. (Note that there is no benefit from cooperation for $a = 1$ (as investments simply add up) and full benefit for $a = 0$.)

There are n, $n \geq 1$, OEMs. For each collected used product, OEM i obtains a positive benefit Δ_i, where $1 \leq i \leq n$. This benefit can result from recycling or remanufacturing activities. Kodak's design for disassembly and recovery enables it to remanufacture at a cost below that of manufacturing (Savaskan et al., 2004). The green remanufacturing program saves Xerox 40%-65% in manufacturing costs through the reuse of parts and materials (Ginsburg, 2001). If remanufactured products are indistinguishable from new ones (as for Fuji Xerox in Japan and Kodak), then the cost savings is directly translated into the benefit per collected item. If remanufactured products are sold at a lower price, then the price difference should be subtracted to obtain the (net) benefit from remanufacturing. In such cases, the net benefit typically remains positive. Indeed, this usually provides the incentive to collect returns. However, even if it is not, it may still be beneficial to collect used items in order to discourage 3rd parties from remanufacturing, e.g., for toner cartridges (Matsumoto and Umeda, 2011), although the benefit per collected item may be harder to be quantified.

5.3.1 Cooperative game: Preliminaries and notation

Let $N = \{1, .., n\}$ denote the set of players, also referred as the grand coalition. Let $v(S)$ denote the characteristic function that assigns a profit value (representing the total amount of transferable utility) to each coalition $S \subseteq N$ ($v(\varnothing) = 0$).

Given a game (N, v), any vector $\pi = (\pi_i)_{i \in N} \in \mathbb{R}^N$ is called a payoff vector, with π_i being the payoff to be given to player $i \in N$ under the condition that cooperation in the grand coalition is reached. A payoff vector $\pi \in \mathbb{R}^n$ is an imputation for the game (N, v) if it is efficient and individually rational, i.e., (i) $\sum_{i \in N} \pi_i = v(N)$;(ii) $\pi_i \geq v(\{i\})$ for all $i \in N$.

The set of imputations of the game (N, v) is denoted by $I(v)$ and it can easily been seen that $I(v)$ is empty iff $v(N) < \sum_{i \in N} v(\{i\})$. The core, $C(v)$, of a game (N, v) is the set

$$C(v) = \left\{ \pi \in \mathbb{R}^N \middle| \sum_{i \in N} \pi_i = v(N), \sum_{i \in S} \pi_i \geq v(S), \forall S \in 2^N \setminus \{\emptyset\} \right\}.$$

The core of a coalitional game requires coalitional rationality in addition to efficiency and individual rationality. It can be interpreted as the set of all allocations of the grand coalition's worth upon which no coalition can improve (Osbourne, 2004).

Finally, we introduce several useful properties of TU-Games. A TU-game is *monotone* if for all $S \subset T \subset N$ it holds that $v(S) \leq v(T)$. Monotonicity can be interpreted as larger coalitions having larger values. A TU-game is *superadditive* if $v(S \cup T) \geq v(S) + v(T)$ for $S \cap T = \emptyset$ and $S, T \subseteq N$. For superadditive games it is efficient for the players to form the grand coalition. Finally a TU-game is convex if for all $i \in N$ and for all $S \subset T \subset N \setminus \{i\}$ we have that $v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T)$. In convex games, the marginal contribution of player i increases as the coalition gets larger. It is known that convex games have a non-empty core (Shapley, 1971). Moreover, convexity of the game is a sufficient condition for the existence of a populational monotonic allocation scheme, PMAS, which is introduced by Sprumont (1990). PMAS reflects the intuition that there is strength in numbers, i.e., an allocation scheme for a game (N, v) is population monotonic if each player's payoff increases as the coalition the player belongs to grows larger (Voorneveld et al., 2003). For a TU-game, (N, v), a scheme $a = (a_{iS})_{i \in S, S \in 2^N \setminus \{\emptyset\}}$ of real numbers is a PMAS of v if (i) $\sum_{i \in S}(a_{iS}) = v(S)$ for all $S \in 2^N \setminus \{\emptyset\}$,(ii) $a_{iS} \leq a_{iT}$ for all $S, T \in 2^N \setminus \{\emptyset\}$ with $S \subset T$ and $i \in S$. Moreover, every core allocation of a convex game can be reached through a PMAS.

5.4 Collection game

In the general collection game that we consider, multiple OEMs invest in collection activities to increase the return rate of used items. To analyze the collection game, we define a general collection situation as a tuple $\langle N, a, L, \{D_i\}_{i\in N}, \{\Delta_i\}_{i\in N}\rangle$.

If a coalition S of firms cooperates, then their joint benefit for any given τ is

$$\left(\tau \sum_{i\in S}(D_i\Delta_i) - (\sum_{i\in S} D_i)^a L\tau^2\right).$$

So the optimal return rate of the coalition S and corresponding joint benefit are given by

$$\tau_S^* = \frac{\sum_{i\in S}(D_i\Delta_i)}{2L\left(\sum_{i\in S} D_i\right)^a}\ , \quad v(S) = \frac{\left(\sum_{i\in S}(D_i\Delta_i)\right)^2}{4L\left(\sum_{i\in S} D_i\right)^a}. \tag{5.1}$$

To have a non-empty set of imputations $I(v)$, and possibly non-empty core $C(v)$, the following condition must hold

$$0 \leq v(N) - \sum_{i\in N} v(\{i\}),$$

which can be rewritten as

$$\left(\left(\sum_{i\in N}(D_i\Delta_i)\right)^2 - \left(\sum_{i\in N} D_i\right)^a \sum_{i\in N}\left(\Delta_i^2 D_i^{(2-a)}\right)\right) \geq 0. \tag{5.2}$$

It is not straightforward to find the combination of benefits, demand rates/shares and economies of scale factor for which (5.2) holds. In what remains of this section, we will therefore derive insights by focusing on special cases. The first is that of $a = 0$, and the reason for considering this case as follows.

It easily follows (See Appendix 5.A.) from (5.2) that the benefit of cooperation decreases with a as expected. Also, if there are no economies of scale (i.e., when $a = 1$), then there is no positive benefit from cooperation. (See Appendix 5.B.) As we will show in Section 5.1, a non-empty set of imputations and a non-empty core does always exist for $a = 0$, i.e., if there are full economies of scale. Although this seems intuitive, it is not straightforward since cooperation in a joint collection scheme implies one identical return ratio for all participants, although their individually optimal return ratios and corresponding investments may differ from each other considerably.

5.4.1 Special Case 1: Full Economies of Scale ($a = 0$)

Clearly, in this case $I(v)$ is non-empty since (5.2) simplifies to

$$\left(\sum_{i \in N}(D_i \Delta_i)\right)^2 - \sum_{i \in N}\left(\Delta_i^2 D_i^2\right) \geq 0 \quad \forall D_i, \Delta_i \; i \in N.$$

The payoff in (5.1) for any coalition $S \subseteq N$ simplifies to

$$v^{[1]}(S) = \frac{\left(\sum_{i \in S} D_i \Delta_i\right)^2}{4L}.$$

Proposition 5.4.1 *For $a = 0$, the collection game is superadditive, monotone and convex.*

Proof. We need to show that for all $S \subset N$, $T \subset N$ and $S \cap T = \emptyset$ it holds that :

$$v^{[1]}(S \cup T) \geq v^{[1]}(S) + v^{[1]}(T).$$

This is equivalent to

$$\frac{\left(\sum_{i \in S \cup T} D_i \Delta_i\right)^2}{4L} \geq \frac{\left(\sum_{i \in S} D_i \Delta_i\right)^2}{4L} + \frac{\left(\sum_{i \in T} D_i \Delta_i\right)^2}{4L},$$

which indeed always holds since $D_i \Delta_i \geq 0$ for all $i \in N$.

Obviously, $\sum_{i \in S} D_i \Delta_i$ is increasing in the number of elements in S for all non-negative D_i and Δ_i. Also, for all non-negative x, x^2 is a monotonically increasing and convex function. Thus, it follows immediately that $(N, v^{[1]})$ is monotone and convex. ■

Since we know that the collection game $(N, v^{[1]})$ is superadditive and convex, we also have that forming the grand coalition is beneficial for the firms and that the core of the game exists. Having proven the existence of the core, it is important to established how it can be reached, i.e., what allocation scheme for the joint benefit is "acceptable" for all players. The next proposition shows that the proportional allocation rule achieves this, under which the benefit received by a firm $i \in N$ is

$$\pi_i^p(v^{[1]}) = \frac{D_i \Delta_i}{\left(\sum_{j \in N} D_i \Delta_i\right)} \times v^{[1]}(N) = \frac{D_i \Delta_i \left(\sum_{j \in N} D_j \Delta_j\right)}{4L}.$$

Proposition 5.4.2 *For $a = 0$, the proportional payoff $\pi^p(v^{[1]})$ is an element of the core.*

Proof. Clearly, $\sum_{i\in N}\pi_i^p(v^{[1]}) = v^{[1]}(N)$ and for all non-empty coalitions S of N it holds that

$$\sum_{i\in S}\pi_i^p(v^{[1]}) = \sum_{i\in S}\frac{D_i\Delta_i\left(\sum_{j\in N}D_j\Delta_j\right)}{4L} \geq \frac{\left(\sum_{i\in S}D_i\Delta_i\right)^2}{4L} = v^{[1]}(S).$$

Thus, $\pi^p(v^{[1]}) \in C(v^{[1]})$. ■

As stated before, every core allocation of a convex game can be reached through a PMAS. Thus, we can also reach proportional allocation rule through a PMAS. This is an important result as it implies that the proportional rule, which is attractive because of its simplicity, satisfies the property of population-monotonicity that requires each player initially present to gain upon the arrival of new players.

Moreover, the proportional rule is stable against the artificial splitting or merging of players. In other words, there is no incentive for the multiple players to artificially present themselves as a single player, for a single player to artificially present himself as a group of players.

5.4.2 Special Case 2: Equal benefits per collected item ($\Delta_i = \Delta$)

Intuitively, a grand coalition is more likely to be formed if all OEMs have similar benefits. In this subsection, we consider the case in which all parties have the same cost benefit per collected item, i.e., $\Delta_i = \Delta$ for every $i \in N$, which would arise if both use similar recycling or remanufacturing technology. Let $v^{[2]}$ denote the characteristic function for this case.

Proposition 5.4.3 *If all players have the same benefit Δ per used item, then the collection game is superadditive, monotone and convex.*

Proof. For all $S \subset N$, $T \subset N$ and $S \cap T = \emptyset$ we have that

$$v^{[2]}(S \cup T) \geq v^{[2]}(S) + v^{[2]}(T),$$

which is equivalent to

$$\frac{\left(\sum_{i\in S\cup T}D_i\right)^{(2-a)}}{4L} \geq \frac{\left(\sum_{i\in S}D_i\right)^{(2-a)}}{4L} + \frac{\left(\sum_{i\in T}D_i\right)^{(2-a)}}{4L}.$$

This inequality always holds, since $D_i \geq 0$ for all $i \in N$ and $0 \leq a \leq 1$, and so $(N, v^{[2]})$ is superadditive.

Obviously, $\sum_{i\in S} D_i$ is increasing in the number of elements in S for all non-negative D_i. Also, for all non-negative x, $x^{(2-a)}$ is monotonically increasing and convex function. Thus, it follows that $(N, v^{[2]})$ is monotone and convex. ■

Similar to Case 1, we consider the proportional rule $\pi^p(v^{[2]})$ for allocating the benefit of cooperation among the players. As a result, firm $i \in N$ receives

$$\pi_i^p(v^{[2]}) = \frac{D_i}{\left(\sum_{j\in N} D_j\right)} \times v^{[2]}(N) = \frac{\Delta^2 D_i \left(\sum_{j\in N} D_j\right)^{(1-a)}}{4L}.$$

Proposition 5.4.4 *For the collection game with equal benefits per collected core, the proportional payoff $\pi^p(v^{[2]})$ is an element of the core.*

Proof. It easily follows that $\sum_{i\in N} \pi^p(v^{[2]}) = v^{[2]}(N)$ and that for all non-empty coalitions S of N it holds that

$$\sum_{i\in S} \pi_i^p(v^{[2]}) = \sum_{i\in S} \frac{\Delta^2 D_i \left(\sum_{j\in N} D_j\right)^{(1-a)}}{4L} \geq \frac{\Delta^2 \left(\sum_{i\in S} D_i\right)^{(2-a)}}{4L} = v^{[2]}(S).$$

Thus, $\pi^p(v^{[2]}) \in C(v^{[2]})$. ■

Similar to Case 1, we can also reach the proportional rule through a PMAS in this case and it is again stable against the artificial splitting or merging players.

5.4.3 Special Case 3: Two players ($|N| = 2$)

The previous two special cases for any number of players have shown that the core exists and can be reached through the proportional rule if there is either full economies of scale or equal benefits for each collected item for all players. Based on this, one expects that the core is more likely to exist for smaller values of a and more similar benefits. To analyze this, we consider the two players case. This will also shed insights on the effect of demand rates/market shares.

Let us define the ratios of demands and cost savings of the two OEMs as $D_2/D_1 = \gamma$ and $\Delta_2/\Delta_1 = \theta$, respectively. From (5.1), we obtain the values for $v(\{1\})$, $v(\{2\})$, and $v(\{1,2\})$ when $S = \{1\}, S = \{2\}$ and $S = \{1,2\}$, respectively. Then, after making necessary simplifications, we get that cooperation is beneficial if

$$0 \leq (1+\theta\gamma)^2 - (1+\gamma)^a(1+\theta^2\gamma^{(2-a)}).$$

Let us define the relative benefit of cooperation (RBC) as the percentage increase in profit of cooperation over the sum of individual profits, i.e.,

$$RBC = \frac{(1+\theta\gamma)^2 - (1+\gamma)^a(1+\theta^2\gamma^{(2-a)})}{(1+\gamma)^a(1+\theta^2\gamma^{(2-a)})} \times 100.$$

Note that the demand and benefit ratios matter rather than the separate values, which allows us to depict the benefit of cooperation graphically in these two ratios for a given value of a. This is illustrated for several values of a in Figure 5.1. This figure confirms our expectations that benefits are larger for smaller values of a and more similar cost savings (θ is close to 1). The effect of demand ratio is more complex. For sufficiently small values of a, so that cooperation is indeed beneficial, if one of the OEM has a smaller benefit per collected item, then it is 'better' if the same firm has a larger market share. This ensures that both players still have an incentive to collect many returns, because of either the larger benefit per collected used item or the large number of collected used items. Indeed, let us define β as the ratio of optimal return rates of OEMs, i.e., $\beta = \tau^*_{\{2\}}/\tau^*_{\{1\}}$. Then, using (5.1), we can write β as, $\beta = \theta\gamma^{(1-a)}$. Figure 5.2 shows that when β is close to 1, the benefit of cooperation is particularly large. In fact, as stated in the next proposition, the RBC is always at its maximum for $\beta = 1$.

Proposition 5.4.5 *For $N = \{1, 2\}$ there is maximum benefit of cooperation if the players' individual optimal return rates are equal, (i.e., $\beta = 1$).*

Proof. For any given γ, and a, let us define

$$f(\theta) = \frac{(1+\theta\gamma)^2 - (1+\gamma)^a(1+\theta^2\gamma^{(2-a)})}{(1+\gamma)^a(1+\theta^2\gamma^{(2-a)})}.$$

Differentiating $f(\theta)$ with respect to θ, we have

$$f'(\theta) = 2\gamma(1+\theta\gamma)\frac{1-\theta\gamma^{1-a}}{(1+\gamma)^a(1+\theta^2\gamma^{(2-a)})^2}.$$

It is clear that $f(\theta)$ is strictly increasing in θ for $\theta < \gamma^{a-1}$ and is strictly decreasing in θ for $\theta > \gamma^{a-1}$. Thus, $\theta = \gamma^{a-1}$ is the global maximizer. By definition, this is equivalent to $\beta = 1$. ■

So, a larger benefit per collected item for one player should ideally be balanced by a larger market share for the other player, where the economies of scale parameter has a moderating effect.

Figure 5.1: Effect of γ and θ on the relative benefit of cooperation for different values of a.

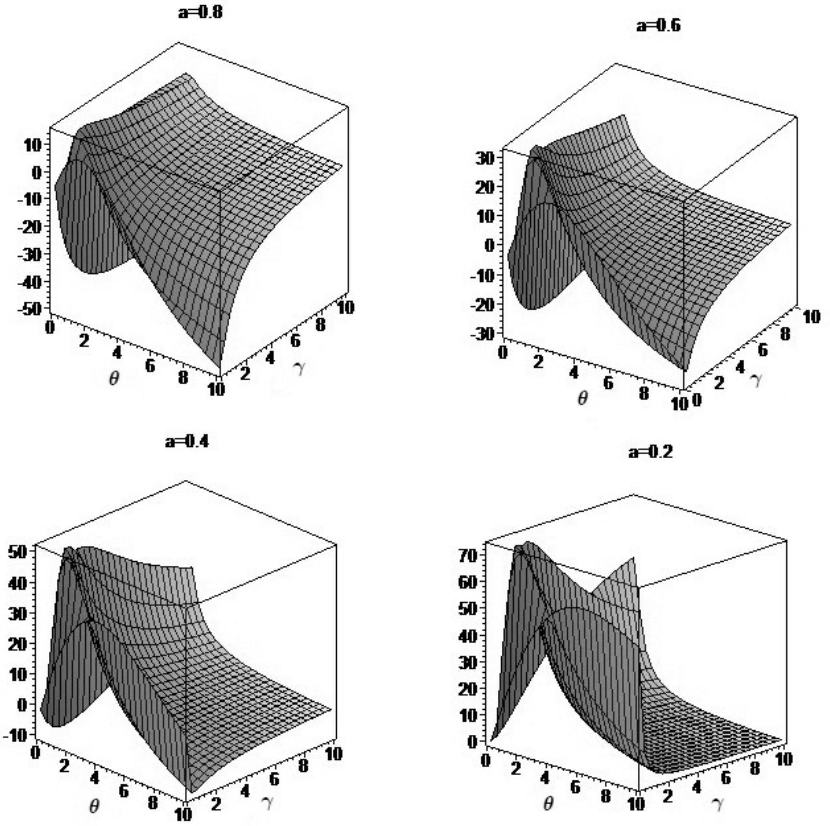
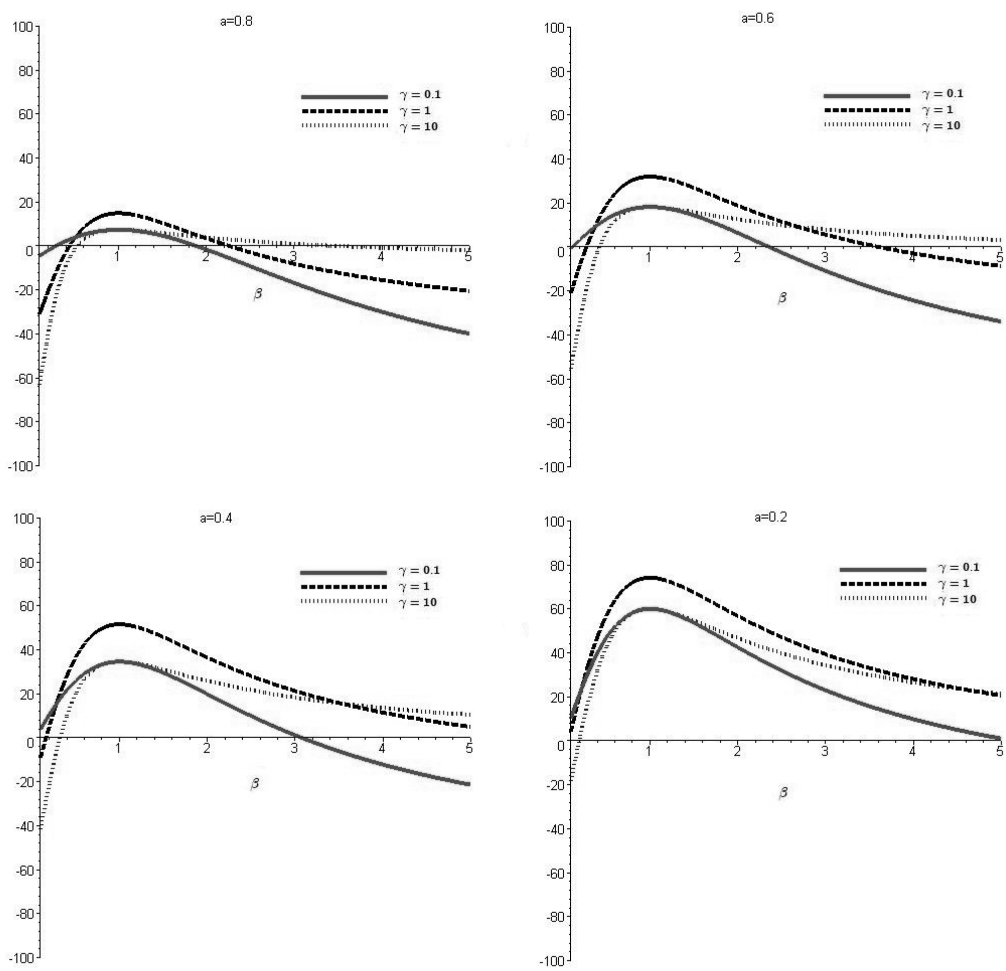


Figure 5.2: Effect of β on the relative benefit of cooperation for different values of a.



5.5 Conclusions

In this chapter, we study supply chain cooperation between OEMs who can jointly invest in activities to collect used products. An important insight is that comparable benefits per collected item provides an important incentive for OEMs to set up a joint collection network. Indeed, for the case with equal benefits per collected used item, which for instance arises if OEMs use the same recovery technology, it is shown that the grand coalition is always formed (i.e., that the core exists) for any number of players and all settings of other parameters in the considered model. The analysis of the two player game shows that if cost asymmetry does exist, then a coalition is more likely to be formed if the OEM with a lower benefit per collected item has a higher market share. So, similar sized market shares do not necessarily motivate OEMs to cooperate, but similar individual return rates do.

An intuitive finding is that the OEMs are more likely to jointly set up a network if there are higher economies of scale (lower value of a). In the absence of economies of scale ($a = 1$), there is obviously no benefit from cooperation. For the case with full economies of scale ($a = 0$), the grand coalition is always beneficial. The latter is not straightforward, as the grand coalition forces all participants to use the same collection network and therefore to settle on the same return rate, although their individual preferences may vary considerably.

For both discussed special cases, either with equal benefits per core or full economies of scale, the cooperation game is monotone, superadditive and convex. From these properties, we obtain that the well-known and easy-to-apply proportional rule is an efficient way to allocate the total benefits of the grand coalition, and that this rule can be reached through a PMAS.

Limitations of this research mainly relate to the modelling of the investment as a function of the return rate and the economies of scale. Although the quadratic relation with the return rate has been proposed in the past and this form is commonly studied in the broader investment literature, other relations can also be considered. Also, economies of scale can be modelled differently from our 'power function' approach. In particular, varying economies of scale for different sets of OEMs could be considered, depending for instance on the geographical overlap of their markets.

Another interesting direction for further research is to simultaneously consider competition between OEMs in selling new products and cooperation in collecting returns. This could be modelled as a two-stage game. Finally, competition of OEMs with 3rd party remanufacturers can also be considered.

Appendix 5.A. Proof of decreasing benefit of cooperation with a

Let

$$\Psi_N(a) = \left(\left(\sum_{i \in N} (D_i \Delta_i) \right)^2 - \left(\sum_{i \in N} D_i \right)^a \sum_{i \in N} \left(\Delta_i^2 D_i^{(2-a)} \right) \right).$$

So we have

$$\frac{\partial \Psi_N(a)}{\partial a} = - \left(\sum_{i \in N} D_i \right)^a \left(\sum_{i \in N} \ln \left(\sum_{i \in N} D_i \right) \left(\Delta_i^2 D_i^{(2-a)} \right) - \sum_{i \in N} \ln D_i \left(\Delta_i^2 D_i^{(2-a)} \right) \right).$$

Since $\ln \left(\sum_{i \in N} D_i \right) > \ln D_i$, for any $D_i > 0$, we have that $\frac{\partial \Psi_N(a)}{\partial a} < 0$ holds. Thus, $\Psi_N(a)$ is strictly decreasing in a. ■

Appendix 5.B. Proof of no benefit from cooperation for the case with $a = 1$

The proof is by induction. For $|N| = 2$ we have

$$\begin{aligned} \Psi_{\{1,2\}}(1) &= (D_1 \Delta_1 + D_2 \Delta_2)^2 - (D_1 \Delta_1^2 + D_2 \Delta_2^2)(D_1 + D_2) \\ &= D_1 D_2 (2 \Delta_1 \Delta_2 - \Delta_1^2 - \Delta_2^2) \\ &= -D_1 D_2 (\Delta_1 - \Delta_2)^2 \leq 0. \end{aligned}$$

Assume that

$$\Psi_{N \backslash \{j\}}(1) = \left(\sum_{i \in N \backslash \{j\}} (D_i \Delta_i) \right)^2 - \left(\sum_{i \in N \backslash \{j\}} D_i \right) \sum_{i \in N \backslash \{j\}} (\Delta_i^2 D_i) \leq 0$$

holds. Then we get

$$
\begin{aligned}
\Psi_N(1) &= \left(\sum_{i\in N\setminus\{j\}}(D_i\Delta_i)+D_j\Delta_j\right)^2-\left(\sum_{i\in N\setminus\{j\}}D_i+D_j\right)\left(\sum_{i\in N\setminus\{j\}}\left(\Delta_i^2D_i\right)+(\Delta_j^2D_j)\right)\\
&= \Psi_{N\setminus\{j\}}(1)+2D_j\Delta_j\sum_{i\in N\setminus\{j\}}(D_i\Delta_i)-D_j\sum_{i\in N\setminus\{j\}}\left(\Delta_i^2D_i\right)-\Delta_j^2D_j\sum_{i\in N\setminus\{j\}}D_i\\
&= \Psi_{N\setminus\{j\}}(1)+D_j\left(\sum_{i\in N\setminus\{j\}}(2\Delta_jD_i\Delta_i)-\sum_{i\in N\setminus\{j\}}\left(\Delta_i^2D_i\right)-\Delta_j^2\sum_{i\in N\setminus\{j\}}D_i\right)\\
&= \Psi_{N\setminus\{j\}}(1)+D_j\left(\sum_{i\in N\setminus\{j\}}D_i(2\Delta_j\Delta_i-\Delta_i^2-\Delta_j^2)\right)\\
&= \Psi_{N\setminus\{j\}}(1)-D_j\left(\sum_{i\in N\setminus\{j\}}D_i(\Delta_j-\Delta_i)^2\right)\leq 0
\end{aligned}
$$

Therefore, if $a=1$, then $v(N)-\sum_{i\in N}v(\{i\})\leq 0$ always holds.■

Chapter 6

Conclusions and directions for further research

The final chapter presents the research findings of the thesis, discusses the conducted research and provides some directions for future research.

Closed loop supply chains and product recovery options have gained increased attention both from industry and academia due to environmental and economical problems such as depletion of natural resources, shortage of key minerals and metals, and waste with toxic materials.

Remanufacturing is an advanced product recovery strategy that is defined as the process of bringing used products to a "like-new" functional state with a warranty to match. This thesis focuses on remanufacturing strategy and aims to provide analytical tools for firms to determine optimal pricing, manufacturing and remanufacturing strategies. To achieve this broad objective with a focused research, the thesis is organized as a combination of research papers. Each paper addresses a specific research question related to different aspects of remanufacturing. In Chapters 2 and 3, we consider a single OEM who performs both manufacturing and remanufacturing. The first aspect is simultaneous consideration of two remanufacturing activities namely, product acquisition management and pricing of new and remanufactured products which is analyzed further in Chapter 2. The second aspect is that of remanufacturing strategy with a capacitated setting and this is addressed in Chapter 3. In Chapters 4 and 5 consider multiple agents in a closed loop supply chain. In Chapter 4, competition between an OEM and an IO is analyzed and in Chapter 5, cooperation between multiple OEMs for returned items collection is considered. In the following paragraphs, the summary of the results and the further research directions are provided for each individual chapter included in the thesis.

Chapter 2 deals with product acquisition management, pricing of new and remanufactured products, and addresses the research question: *What is the optimal quantity and quality of the acquired cores and the price at which new and remanufactured products are sold?* the main contribution of the chapter is simultaneous consideration of two key remanufacturing activities namely, product acquisition management and pricing of the new and remanufactured products. In the problem formulation, there exists an OEM that manufactures new items as well as remanufactures returned items where returned items are classified into different types. The OEM decides both on how much to purchase from each return type (with the acquisition prices offered) and under which price to sell new and remanufactured items, where consumers have lower willingness to pay for the remanufactured items. In Chapter 2, an algorithm is derived to find the closed-form optimal solution. The main findings in Chapter 2 are as follows. In line with the results in literature, the profit per new item only depends on the manufacturing cost, and instead of having an equal profit per remanufactured item from all return types, the profit per item is higher if the total cost for acquisition and remanufacturing is lower. The main managerial implication is that although practitioners may find it intuitive and/or fair to apply equal profits, doing so is suboptimal. Possible future research directions are to analyze two or multiple period settings where the available number of returns in a period depends on the product sales in the prior period, to consider different selling prices for different core types, and to include a competing third party remanufacturer.

In Chapter 3, the following research questions are addressed: *(i) What is the impact of remanufacturing on the optimal capacity and production decisions? (ii) If remanufacturing is either more costly or more capacity intensive, can it still be profitable?* The main contribution of this chapter is to analyze the combined effects of cost and capacity usage of remanufacturing on production and capacity decision and on the profitability of remanufacturing. The analysis is done for a two-period model with capacity and manufacturing decisions in the first period and manufacturing and remanufacturing decisions in the second period for three different scenarios regarding the relative cost and capacity requirement of remanufacturing. First, the most preferred and realistic scenario, which is less costly and less capacity intensive remanufacturing, is considered. Closed-form solutions are derived for all capacity and production decisions and used to conduct a sensitivity analysis. The main findings of this chapter are as follows. In some cases, remanufacturing only replaces manufacturing to reduce cost leaving the total production in the second period unchanged. However, for other cases, remanufacturing does increase the total production quantity. A particularly interesting finding is that the availability of the less capital intensive remanufacturing option sometimes leads to an increased capital investment. In such cases, the additional investment is made to increase production in period 1 and thereby the number of returns in period 2. In other cases, with surplus capacity in the first period, introducing the remanufacturing option does lead to a reduction in the capacity investment.

The scenarios in which remanufacturing is either more costly or more capacity intensive than manufacturing are analyzed in a numerical study. It turns out that the scenario where remanufacturing is more costly is by far the least beneficial and remanufacturing can retain considerable benefits if it is more capacity intensive, but only if the cost and market conditions are such that there is sufficient capacity in the latter stage of the life-cycle (period 2). One possible extension can be to consider a stochastic return rate due to the market uncertainty. Also, more than two-periods would allow a more accurate description of a product life-cycle. Inclusion of other players such as collectors and/or OEMs and inroducting competition for collecting cores and/or selling (re)manufactured products can be considered.

Chapter 4 analyzes the effect of competition with an IO on the remanufacturing strategy of an OEM in a two-period setting and addresses the following research questions: *(i) What are the optimal production and pricing strategies of the parties in a supply chain in the case of competition? (ii) What is the effect of competition on the remanufacturing strategy?* The main contribution of the model to the existing literature is to model competition between the parties for collecting cores, next to the competition for market share. The existence of a unique Nash equilibrium between the parties in period 2 is determined and the optimal acquisition prices and (re)manufacturing quantities are found. Moreover, the optimal manufacturing quantity for the OEM for period 1 is found. In the main part of the study, the case in which the new and remanufactured products are sold for the same price is analyzed. Several managerial insights for this case are obtained through sensitivity analyses. First of all, it is determined that OEM's acquisition price only depends on its cost structure, not on IO's acquisition price which implies that the OEM does not compete for collecting cores. It is also found that remanufacturing is a profitable option for the OEM when it has a dominance in collecting the available cores and remanufacturing leads cost savings. Moreover, when the cost benefit of remanufacturing diminishes or the IO improve its ability (compared to the OEM) to collect the available cores, the OEM manufactures less in the first period to protect its overall market share.

Afterwards, the case where consumers have lower willingness to pay for the remanufactured products is considered and the effects of the parameters on the optimal production quantities and acquisition prices are determined to be similar with the case of indistinguishable remanufactured products. As expected, when remanufactured products are values less than new products, the acquisition prices offered both by the OEM and IO decreases, and remanufacturing becomes less profitable overall.

In Chapter 4, it is assumed that the consumers' willingness to pay for the products is uniformly distributed which leads to linear inverse demand functions. In further research, other distributions can also be used for modeling consumers' willingness to pay for the products. Other possible research areas are to consider random demand and competition between more than two parties can be considered. Finally, capacity restrictions can be included.

Chapter 5 studies supply chain cooperation among OEMs who can jointly invest in activities to collect used products and addresses the research question *When does cooperation lead to joint cost-savings, and how should these savings be allocated among the firms?* Using cooperative game theory we find that comparable benefits per collected item provides an important incentive for OEMs to set up a joint collection network. Indeed, for the case with equal benefits per collected used item, which for instance arises if OEMs use the same recovery technology, we show that the grand coalition is always formed for any number of players and all settings of other parameters in the considered model. The analysis of the two player game shows that if cost asymmetry does exist, then a coalition is more likely to be formed if the OEM with a lower benefit per collected item has a higher market share. So, similar sized market shares do not necessarily motivate OEMs to cooperate, but similar individual return rates do.

An intuitive finding is that the OEMs are more likely to jointly set up a network if there are higher economies of scale. In the absence of economies of scale, there is obviously no benefit from cooperation. For the case with full economies of scale, the grand coalition is always beneficial. The latter is not straightforward, as the grand coalition forces all participants to use the same collection network and therefore to settle on the same return rate, although their individual preferences may vary considerably. For both discussed special cases, either with equal benefits per core or full economies of scale, the cooperation game is monotone, superadditive and convex. From these properties, it is obtained that the well-known and easy-to-apply proportional rule is an efficient way to allocate the total benefits of the grand coalition. An interesting further research direction is that varying economies of scale for different sets of OEMs could be considered, depending for instance on the geographical overlap of their markets. Another interesting direction for further research is to simultaneously consider competition between OEMs in selling new products and cooperation in collecting returns. This could be modelled as a two-stage game. Finally, competition of OEMs with 3rd party remanufacturers can also be considered.

To conclude, the starting point of our research was to provide analytical tools to firms for optimal strategies in hybrid manufacturing/remanufacturing systems. To achieve this goal, we analyze different scenarios in each chapter of this thesis by building mathematical model and determine an OEM's optimal core acquisition, production and pricing strategy and investigate profitability of remanufacturing. The main findings of this thesis are as follows. If there are different return types acquired by the OEM, it is suboptimal to apply equal profits to remanufactured items. In the case of capacitated production setting, remanufacturing can be used to reduce production cost or increase production. Also, if remanufacturing requires more capacity or is more costly than manufacturing, it may still be profitable for an OEM. In the case of competition with an IO the ability of core collection is vital for an OEM for profitable manufacturing. Finally, cooperation with other OEMs in product returns channel is profitable for an OEM if the OEMs have similar cost benefits (similar recovery technologies). In this thesis we consider different scenarios and as described above further research directions are mainly to consider these scenarios simultaneously. For instance, cooperation between the OEMs in the product returns channel while competing in the sales market and competition between an OEM and IO in capacitated setting with different return types can be considered.

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Samenvatting (Summary in Dutch)

Closed loop supply chains en de mogelijkheid tot hergebruik van producten zijn in toenemende mate onderwerp van onderzoek in zowel de industrie als de academische wereld. De populariteit van deze onderzoeksgebieden kan gemakkelijk verklaard worden door de economische crisis en de toegenomen aandacht voor duurzaam ondernemen. De uitputting van natuurlijke hulpbronnen, het tekort aan belangrijke mineralen en metalen en het afvalvraagstuk van giftige stoffen geven een directe aanleiding om bepaalde grondstoffen en producten opnieuw te gebruiken. Remanufacturing is een geavanceerde product recovery strategie waarin gebruikte producten een behandeling ondergaan zodat ze gegarandeerd weer "zo goed als nieuw" zijn. Dit proefschrift richt zich op remanufacturing en heeft tot doel om analytische hulpmiddelen te ontwikkelen waarmee bedrijven optimale prijsstelling-, productie- en remanufacturing-strategieën kunnen bepalen. Om met toegespitst onderzoek aan deze brede doelstelling te kunnen voldoen, zijn verscheidende onderzoeksartikelen in dit proefschrift gebundeld. Elk artikel vormt een hoofdstuk en richt zich op een specifieke vraagstelling die betrekking heeft op n of meerdere aspecten van remanufacturing.

Hoofdstuk 2 behandelt zowel het inkoopmanagement van gebruikte producten als de prijsstelling van nieuwe en gerecirculeerde producten, en tracht de volgende onderzoeksvraag te beantwoorden: Wat zijn de optimale hoeveelheid en kwaliteit van de in te zamelen gebruikte artikelen en wat is de prijs waartegen nieuwe en gerecirculeerde producten moeten worden verkocht? De toegevoegde waarde van dit hoofdstuk bestaat uit de gelijktijdige behandeling van twee belangrijke remanufacturing activiteiten, namelijk het inkoopmanagement van gebruikte goederen en de prijsstelling van nieuwe en gerecirculeerde producten. In dit hoofdstuk wordt een OEM (Original Equipment Manufacturer) beschouwd die nieuwe items produceert en bovendien geretourneerde artikelen recirculeert. De geretourneerde artikelen zijn onderverdeeld in verschillende categorieën, afhankelijk van de staat waarin ze verkeren. De OEM bepaalt de hoeveelheid aan te kopen gebruikte artikelen in elke categorie, de bijbehorende inkoopprijzen en de verkoopprijzen van nieuwe en gerecirculeerde artikelen. Hierbij is het van belang dat verondersteld wordt dat consumenten bereid zijn meer te betalen voor nieuwe dan voor gerecirculeerde goederen. In Hoofdstuk 2 wordt een algoritme gentroduceerd waarmee de optimale oplossing in gesloten vorm bepaald kan worden. De voornaamste bevindingen van dit hoofdstuk zijn als volgt. Overeenkomstig met bestaande resultaten in de literatuur hangt de winst per nieuw geproduceerd item alleen af van de productiekosten. De winst per gerecirculeerd item is niet gelijk voor elke categorie, maar is hoger naarmate de totale kosten voor inkoop en remanufacturing lager zijn. Hoewel men het in de praktijk vanzelfsprekend dan wel eerlijk vindt om nieuwe en gebruikte goederen met dezelfde winst te verkopen, blijkt uit Hoofdstuk 2 dat dit suboptimaal is. In toekomstig onderzoek zou men kunnen kijken naar een model dat meerdere periodes omvat en waar het aantal gebruikte producten dat beschikbaar is voor remanufacturing afhangt van het aantal verkochte items in de voorgaande periode. Daarnaast zou men voor elk type gebruikt product de verkoopprijs afzonderlijk kunnen vaststellen, of de mogelijkheid van een extra remanufacturer in ogenschouw kunnen nemen. In Hoofdstuk 3 komen de volgende onderzoeksvragen aan bod: (i) Wat is de invloed van remanufacturing op de optimale capaciteits- en productiebeslissingen? (ii) Blijft remanufacturing rendabel als het duurder dan wel capaciteitintensiever wordt? De belangrijkste bijdrage van dit hoofdstuk aan de bestaande literatuur is het analyseren van het gecombineerde effect van de kosten en het capaciteitsgebruik van remanufacturing op de productie- en capaciteitsbeslissingen enerzijds en op de rentabiliteit van remanufacturing anderzijds. De analyse is uitgevoerd voor een twee-perioden model; in de eerste periode worden de productiecapaciteit en het productieaantal voor periode n bepaald en in de tweede periode het productieaantal voor periode twee en het aantal te recirculeren producten. Dit wordt gedaan voor drie verschillende scenarios, waarin de relatieve kosten en capaciteitsbehoeftes variëren. Als eerste wordt het scenario dat het meest realistisch is en dat de meeste voorkeur geniet beschouwd. In dit scenario is remanufacturing minder kostbaar en minder capaciteitsintensief dan nieuwe productie. Voor alle capaciteits- en productiebeslissingen worden oplossingen in gesloten vorm afgeleid. Deze oplossingen worden gebruikt om een gevoeligheidsanalyse uit te voeren. De belangrijkste bevindingen van dit hoofdstuk zijn als volgt.

In sommige gevallen neemt remanufacturing de plaats in van nieuwe productie zodat de kosten afnemen terwijl de totale productie in de tweede periode (nieuw plus remanufacturing) ongewijzigd blijft. Er zijn echter ook gevallen waarin remanufacturing de totale productiehoeveelheid verhoogt. Bijzonder interessant is dat remanufacturing, hoewel minder kapitaalintensief, soms leidt tot een verhoogde kapitaalinjectie. In dergelijke gevallen wordt de extra investering gedaan om de productie in periode n en daarmee het aantal geretourneerde producten in periode twee te verhogen. Er zijn echter ook gevallen waarin sprake is van overcapaciteit in de eerste periode en waarin de invoering van remanufacturing wel degelijk tot een daling van de capaciteitsinvestering leidt. De scenario's waarin remanufacturing duurder of capaciteitsintensiever is dan nieuwe productie worden geanalyseerd in een numerieke studie. Het blijkt dat het scenario waarin remanufacturing duurder is veruit het minst gunstig is. Remanufacturing leidt daarentegen tot aanzienlijke kostenvoordelen wanneer het relatief capaciteitsintensief is, mits de kosten en marktcondities zo zijn dat er voldoende capaciteit is gedurende de tweede periode van de levenscyclus van het product. De invoering van een stochastisch retourneringspercentage om onzekerheid op de markt te modelleren zou een interessante uitbreiding van het model zijn voor toekomstig onderzoek. Daarnaast kan de levenscyclus van het product waarheidsgetrouwer gemodelleerd worden door deze in meer dan twee periodes onder te verdelen. Tenslotte zou men het aantal spelers op de markt kunnen uitbreiden, variërend van het aantal inzamelaars van gebruikte producten tot het aantal OEMs, en zou concurrentie tussen inzamelaars van gebruikte producten en / of de verkoop van nieuwe dan wel gerecirculeerde goederen kunnen worden onderzocht. Hoofdstuk 4 analyseert het effect dat concurrentie met een Independent Operator (IO, een onafhankelijk ondernemer) heeft op de remanufacturing-strategie van een OEM in een twee-perioden model. De volgende onderzoeksvragen komen aan bod: (i) Wat zijn de optimale productie- en prijsstellingstrategieën van de spelers in een supply chain wanneer er sprake is van concurrentie? (ii) Wat is het effect van concurrentie op de remanufacturing-strategie? De belangrijkste bijdrage van het model aan de bestaande literatuur is dat er, naast de concurrentie om marktaandeel, sprake is van concurrentie bij het opkopen van gebruikte producten. In dit hoofdstuk wordt het unieke Nash-evenwicht tussen de partijen in periode twee bepaald evenals de optimale acquisitieprijzen, de te produceren en de te recirculeren hoeveelheden. Daarnaast wordt de optimale productiehoeveelheid van de OEM in periode n berekend. In dit hoofdstuk staat het scenario centraal waarin de nieuwe en gerecirculeerde producten tegen dezelfde prijs worden verkocht. Door middel van gevoeligheidsanalyses worden voor dit scenario verschillende bestuurlijke inzichten verkregen. Allereerst wordt aangetoond dat de inkoop van de OEM alleen afhangt van de kostenstructuur en niet van de inkoopprijs van de IO. Dit impliceert dat de OEM niet concurreert op de inkoop van gebruikte goederen. Daarnaast wordt aangetoond dat remanufacturing rendabel is als de OEM een dominerende positie heeft op de markt van gebruikte producten en als remanufacturing tot kostenbesparingen leidt. Wanneer het kostenvoordeel van recirculatie afneemt of wanneer de IO ten opzichte van de OEM beter in staat is om de beschikbare gebruikte goederen in te zamelen, dan produceert de OEM bovendien minder in de eerste periode om zijn totale marktaandeel te beschermen. Na de analyse van het geval waarin nieuwe en gebruikte goederen evenveel opleveren, wordt een scenario bekeken waarin consumenten minder willen betalen voor gerecirculeerde producten. Het blijkt dat de effecten van de parameters op de optimale productiehoeveelheden en acquisitieprijzen vergelijkbaar zijn met het geval waarin gerecirculeerde producten niet te onderscheiden zijn van nieuwe producten. Logischerwijs dalen de verkoopprijzen van zowel de OEM als de IO voor gerecirculeerde goederen wanneer deze minder waard zijn dan nieuwe producten. Tegelijkertijd neemt daarmee de winstgevendheid van remanufacturing af. In Hoofdstuk 4 wordt aangenomen dat de bereidheid van consumenten om te betalen uniform verdeeld is, wat leidt tot lineaire inverse vraagfuncties. In toekomstig onderzoek zou men andere verdelingsfuncties kunnen introduceren om de bereidheid tot betalen van consumenten te modelleren. Daarnaast zou men gevallen met stochastische vraag of concurrentie tussen meerdere partijen kunnen onderzoeken. Tenslotte kunnen capaciteitsbeperkingen in het model worden opgenomen. In Hoofdstuk 5 wordt een samenwerkingsverband tussen OEMs in een supply chain bestudeerd, waarbij de OEMs gezamenlijk kunnen investeren in activiteiten om gebruikte producten in te zamelen. De studie in dit hoofdstuk is gericht op de volgende onderzoeksvraag: Wanneer leidt samenwerking tot gezamenlijke kostenbesparingen, en hoe moeten deze besparingen onderling worden verdeeld? Met behulp van coperatieve speltheorie leiden we af dat gelijke opbrengsten per ingezameld item OEMs een belangrijke prikkel geven om een gezamenlijk inzamelnetwerk op te zetten. Er is sprake van gelijke opbrengsten als OEMs bijvoorbeeld gebruik maken van dezelfde hersteltechnologie. Voor dit soort gevallen tonen we aan dat de grand coalition altijd wordt gevormd, onafhankelijk van het aantal spelers of de parameterwaarden in het onderliggende model. Dit wil zeggen dat alle OEMs deelnemen aan het samenwerkingsverband. Uit de analyse van het geval met twee spelers volgt dat als de kostenstructuur van de OEMs niet symmetrisch is, er een grotere kans op een coalitie is indien de OEM met een lagere opbrengst per ingezameld item een hoger marktaandeel heeft. Een gelijke verdeling van marktaandelen stimuleert OEMs dus niet noodzakelijkerwijs tot samenwerking, terwijl vergelijkbare individuele retourneringspercentages dat wel doen. Een intutieve bevinding is dat de OEMs eerder geneigd zijn een gezamenlijk netwerk op te zetten als de schaalvoordelen groter zijn. Bij het ontbreken van schaalvoordelen is er uiteraard geen reden tot samenwerking. Indien er sprake is van volledige schaalvoordelen dan is de grand coalition altijd voordelig. Dit laatste is niet voor de hand liggend, omdat de grand coalition alle deelnemers dwingt om gebruik te maken van hetzelfde inzamelnetwerk en hetzelfde retourneringspercentage te accepteren, terwijl hun individuele voorkeuren aanzienlijk kunnen verschillen. Voor beide speciale gevallen, dat wil zeggen met gelijke opbrengsten per ingezameld product of met volledige schaalvoordelen, is het coperatieve spel monotoon, superadditief en convex. Met behulp van deze eigenschappen kan worden afgeleid dat de bekende en eenvoudig toe te passen evenredigheidsregel een efficiënte manier vormt om de totale baten van de grand coalition te verdelen. Als interessante richting voor vervolgonderzoek zou men kunnen kijken naar uiteenlopende schaalvoordelen voor verschillende combinaties van OEMs, die bijvoorbeeld afhangen van de geografische overlap van hun markten. Ander mogelijk vervolgonderzoek zou zich kunnen richten op een model dat tegelijkertijd concurrentie tussen OEMs bij de verkoop van producten en samenwerking bij het inzamelen van gebruikte producten beschouwt. Dit zou gemodelleerd kunnen worden als een spel dat uit twee opeenvolgende fases bestaat. Tenslotte kan de concurrentie van OEMs met onafhankelijke remanufacturers ook worden overwogen.

Samenvattend kunnen we stellen dat het uitgangspunt van dit onderzoek was om analytische instrumenten te ontwikkelen waarmee bedrijven optimale strategieën in hybride productie- / remanufacturing-systemen kunnen bepalen. Om dit doel te bereiken hebben we in elk hoofdstuk van dit proefschrift aan de hand van wiskundige modellen verschillende scenarios geanalyseerd en hebben we de optimale productie-, remanufacturing- en prijsstellingstrategie van een OEM bepaald. De belangrijkste bevindingen van dit proefschrift zijn als volgt. Indien er verschillende categorieën gebruikte producten worden ingezameld door de OEM, dan is het suboptimaal om alle categorieën met gelijke winst te verkopen. In het geval van een gelimiteerde productiecapaciteit kan remanufacturing worden gebruikt om de productiekosten te verminderen of om de productie te verhogen. Zelfs als remanufacturing meer capaciteit in beslag neemt of duurder is dan nieuwe productie, kan het nog steeds winstgevend zijn voor een OEM. Wanneer er sprake is van concurrentie met een IO dan is het vermogen van een OEM om gebruikte goederen in te zamelen van wezenlijk belang om de productie winstgevend te laten zijn. Tot slot is samenwerking met andere OEMs in het retourneringskanaal winstgevend als de deelnemende OEMs vergelijkbare kostenvoordelen hebben (bijvoorbeeld door het gebruik van vergelijkbare hersteltechnologieën). In dit proefschrift beschouwen we verschillende scenario's en zoals reeds beschreven liggen mogelijkheden voor toekomstig onderzoek vooral in het gelijktijdig analyseren van verschillende scenarios. Zo zou men kunnen kijken naar een setting waarin OEMs een samenwerkingsverband aangaan in het retourneringskanaal terwijl ze concurreren op de afzetmarkt. Een andere interessante optie is om onderzoek te doen naar concurrentie tussen een OEM en een IO als er sprake is van verschillende categorieën gebruikte goederen en een gelimiteerde productiecapaciteit.